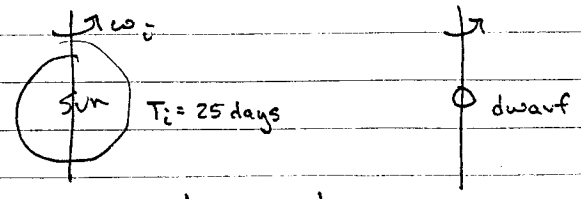
55.



solid ball $\frac{2}{5}MR^2$

$R_{sun} = 6.96 \times 10^8$ m =

$R_{Earth} = 6370$ km

$$L_{init} = L_{final}$$

$$I_{init}\,\omega_{init} = I_{final}\,\omega_{final}$$

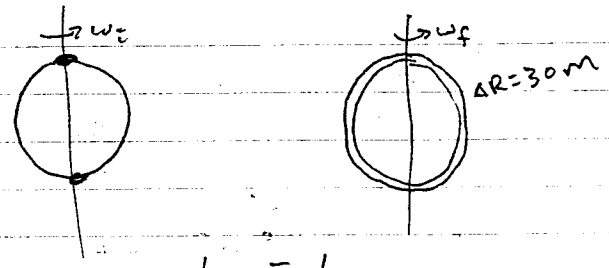
$$\left(\tfrac{2}{5} M R_{init}^2\right)\left(\frac{2\pi}{T_i}\right) = \left(\tfrac{2}{5} M R_{fin}^2\right)\left(\frac{2\pi}{T_f}\right)$$

$$\frac{R_{init}^2}{T_i} = \frac{R_{final}^2}{T_f}$$

$$\frac{(6.96 \times 10^8\ \text{m})^2}{25\ \text{days}} = \frac{(6370 \times 10^3\ \text{m})^2}{T_f}$$

$$T_f = 2.1 \times 10^{-3}\ \text{days}$$
$$= 5.0 \times 10^{-2}\ \text{hrs}$$
$$= 3.0\ \text{min}$$

63.



$I_{solid\ sphere} = \frac{2}{5} MR^2$

$$L_{init} = L_{final}$$

$$I_{init}\,\omega_{init} = I_{final}\,\omega_f$$

There are a couple of approaches that one can take to get the moments of inertia.

① Assume that the I_{final} can be calculated by just allowing the Earth to get bigger by 30m

$$\left(\tfrac{2}{5} M R_0^2\right)\omega_{init} = \left(\tfrac{2}{5} M R_{final}^2\right)\omega_{final}$$

$$R_0^2 \frac{2\pi}{T_{init}} = R_f^2 \frac{2\pi}{T_{final}}$$

$$\frac{T_{final}}{T_{init}} = \left(\frac{R_{final}}{R_{init}}\right)^2 = \left(\frac{R_0 + \Delta R}{R_0}\right)^2 = \left(1 + \frac{\Delta R}{R_0}\right)^2$$

Use binomial expansion to evaluate.

$$\frac{T_{final}}{T_{init}} \simeq 1 + 2\frac{\Delta R}{R_0}$$

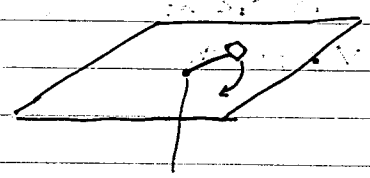
$$\frac{T_{init} + \Delta T}{T_{init}}$$

$$1 + \frac{\Delta T}{T_{init}} \simeq 1 + 2\frac{\Delta R}{R}$$

$$\frac{\Delta T}{T_{init}} \simeq 2\frac{\Delta R}{R} = 2\,\frac{30\ \text{m}}{6370 \times 10^3\ \text{m}}$$

$$\Delta T = 9.4 \times 10^{-6}\ \text{days} = 0.82\ \text{sec}$$

10.48

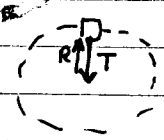


$m = 0.4\ kg$

Initially $v_i = 4\ m/s$ $R_i = 0.5\ m$

what is ~~breaking speed~~ radius when cord breaks at $T = 60\ N$

top view



torque = 0 because $\vec{r} \nparallel \vec{\text{Force}}$

i.e. Ang Mom conserved

$$L_{init} = L_{final}$$
$$I_i \omega_i = I_f \omega_f$$
$$m R_i^2 \omega_i = m R_f^2 \omega_f$$

$V = R\omega$

$$\cancel{m} R_i v_i = \cancel{m} R_f v_f$$

Forces are governed by $F = ma$

So $m a_{rad} = T$

$m\, v^2/R = T$

At the breaking point $T \to 60$ Newtons

$v \to v_f$

$R \to R_f$

[$v_f = \frac{R_i v_i}{R_f}$ from above

$$m \frac{\left(\frac{R_i v_i}{R_f}\right)^2}{R_f} = T$$

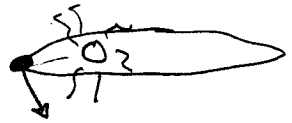
$$m \frac{R_i^2}{R_f^2} v_i^2 = T R_f$$

$$m R_i^2 v_i^2 = T R_f^3$$

$$(0.4)(0.5)^2 4^2 = 60 R_f^3$$

$$R_f = 0.30\ m$$

12-46



$$L_{perihelion} = L_{aphelion}$$

$$r_p p_p = r_a p_a$$

$$r_p m v_p = r_a m v_a$$

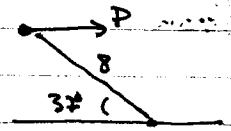
$$(1.471 \times 10^{11})(3.027 \times 10^{4}) = (1.521 \times 10^{11}) v_a$$

$$v_a = 2.927 \times 10^{4} \text{ m/s}$$

10.23 $\vec{L} = \vec{r} \times \vec{p}$

$|\vec{L}| = L = r_\perp p$

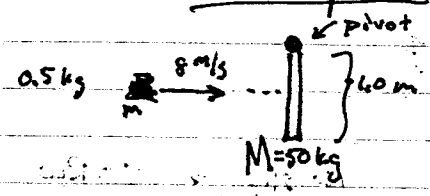
$r_\perp$ ← impact parameter



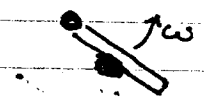
impact parameter $= r_\perp = (8\,m)\sin 37°$
$= 4.8\,m$

$L = 4.8\,(0.6\,kg \cdot 12\,m/s)$
$= 34.6\ kg\,m^2/s$

10.27 Before top view



After top view



Conservation of Ang Momentum

$L_{before} = L_{after}$

$r_\perp mV = I_{door}\,\omega + I_{mud}\,\omega$

$0.5(0.5)8 =$

$I_{door} = \frac{1}{3} M a^2 = \frac{1}{3}\,50\,(1)^2$
$= 16.7\ kg\,m^2$

$I_{mud} = MR^2 = (0.5)(0.5)^2$
(point obj) $= 0.125\ kg\,m^2$

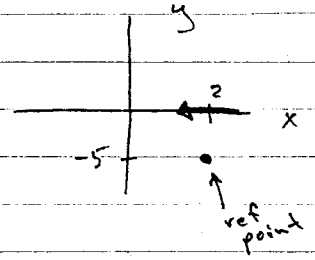
$2 = (I_{door} + I_{mud})\,\omega$

$2 = (16.7 + 0.125)\,\omega$

$\omega = 0.119\ rad/sec$

The moment of Inertia of mud is only 8% effect.

c)



$$\vec{r}_{\substack{\text{wrt}\\ \text{ref}\\ \text{point}}} = (x_0 - 2 - \tfrac{1}{2}t^4)\,\hat{i} + 5\,\hat{j} + 0\,\hat{k}$$

$$\vec{p} = -6t^3\,\hat{i} + 0\,\hat{j} + 0\,\hat{k}$$

$$\vec{F} = -18\,\hat{i} + 0\,\hat{j} + 0\,\hat{k}$$

$$\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_0 - 2 - \frac{1}{2}t^4 & 5 & 0 \\ -6t^3 & 0 & 0 \end{vmatrix} = \hat{i}\,(0) - \hat{j}\,(0) + \hat{k}\left[-(5)(-6t^3)\right]$$

$$= 30t^3\,\hat{k}$$

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_0 - 2 - \frac{1}{2}t^4 & 5 & 0 \\ -18 & 0 & 0 \end{vmatrix} = \hat{i}\,(0) - \hat{j}\,(0) + \hat{k}\left[-(-18)5\right]$$

$$= 90\,\hat{k}$$

51.

$I_i = 6\ \text{kg m}^2$ $\qquad$ $I_f = 2\ \text{kg m}^2$

$\omega_i = 1.2\ \text{rev/s}$ $\qquad$ $\omega_f = ?$

a) $L_{init} = L_{final}$

$I_{init}\,\omega_i = I_f\,\omega_f$

$6\,(1.2) = 2\,\omega_f$

$\omega_f = 3.6\ \text{rev/s}$

b) $\omega_i = 1.2\ \text{rev/sec} = (1.2)2\pi\ \text{rad/sec} = 7.54\ \text{rad/s}$

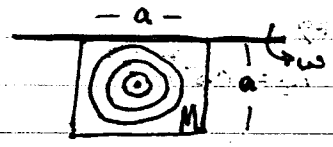
$\omega_f = 3.6\ \text{rev/sec} = 3.6(2\pi)\ \text{rad/sec} = 22.61\ \text{rad/s}$

c) $KE_i = \frac{1}{2} I_i \omega_i^2 = \frac{1}{2}(6)(7.54)^2 = 171\ \text{J}$

$KE_f = \frac{1}{2} I_f \omega_f^2 = \frac{1}{2}(2)(22.61)^2 = 511\ \text{J}$

Work done by his arms

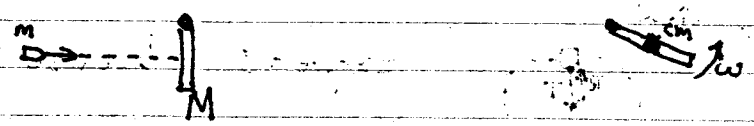
12.49



$M = 2.2$ kg $a = 0.2$ m

bullet: $m = 5\times10^{-3}$ kg $V = 300$ m/s

a) side view



Collision → conserv. momentum

$L_{before} = L_{after}$

Assume bullet stops in target

$$r_\perp mV = I_{board}\,\omega + I_{bullet}\,\omega$$
$$= (I_{board} + I_{bullet})\,\omega$$

$$I_{board} = \tfrac{1}{3} M a^2$$
$$= \tfrac{1}{3}\, 2.2\,(0.2)^2$$
$$= 0.0293 \text{ kg m}^2$$

$$I_{bullet} = m R^2 =$$
$$= 5\times10^{-3}\,(0.1)^2$$
$$= 5\times10^{-5} = \text{very small}$$

$$0.1\,(5\times10^{-3})\,300 \doteq (0.0293)\,\omega$$
$$\omega = 5.12 \text{ rad/s}$$

b) Max height → board is rotating initially.

$$\tfrac{1}{2} I\omega^2 = mgh$$
$$\tfrac{1}{2}(0.0293)(5.12)^2 = 2.2\,(9.8)\,h$$
$$h = 0.0178 \text{ m}$$ – hgt cm raises up.

c) Collecting previous equations

$r_\perp mV = I_{board}\,\omega$ and $\tfrac{1}{2} I\omega^2 = mgh$

$$\omega = \frac{r_\perp mV}{I}$$

$$\tfrac{1}{2} I \left(\frac{r_\perp mV}{I}\right)^2 = mgh$$

$$\tfrac{1}{2}\,\frac{r_\perp^2 m^2 V^2}{I} = mgh$$

$$\tfrac{1}{2}\,\frac{(0.1)^2 (5\times10^{-3})^2 V^2}{0.0293} = 2.2\,(9.8)\,0.2$$

$$(4.27\text{-}6)\,V^2 = \quad \rightarrow \quad V = 1005 \text{ m/s}$$