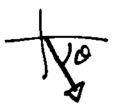
4.6 Electron's position

$\vec{r} = 3t\,\hat{i} - 4t^2\,\hat{j} + 2\,\hat{k}$

a) $\vec{v} = \frac{d\vec{r}}{dt} = 3\hat{i} - 8t\,\hat{j}$

b) at $t=2$ $\quad \vec{v} = 3\hat{i} - 16\,\hat{j}$

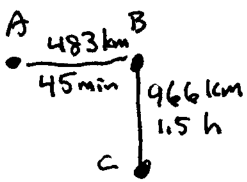
c,d)



$|\vec{v}| = v = \sqrt{3^2+16^2} = 16.3$ m/s

$\tan\theta = -\frac{16}{3} \rightarrow \theta = -79.4^\circ$

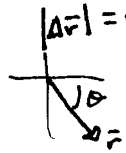
4.8



a) Displacement vector

$\Delta\vec{r} = 483\,\hat{i} - 966\,\hat{j}$ (km)

$|\Delta\vec{r}| = \sqrt{483^2+966^2} = 1080$ km



$\tan\theta = \frac{-966}{483}$

$\rightarrow \theta = -64.1^\circ$

c) $\overrightarrow{\text{Average Velocity}} = \frac{\text{net } \overrightarrow{\text{displacement}}}{\text{time}}$

$= \frac{483\,\hat{i} - 966\,\hat{j}}{2.25\,h}$

$= 215\,\hat{i} - 429\,\hat{j}$ (km/hr)

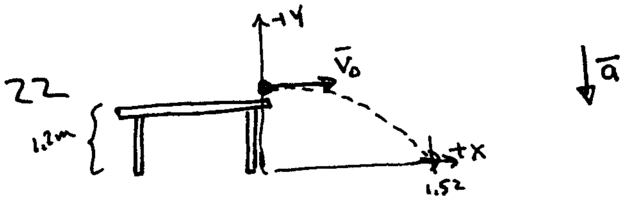
$|\vec{v}_{AVG}| = v_{AVG} = \sqrt{215^2+429^2} = 480$ km/hr

$\tan\theta = \frac{-429}{215} \rightarrow \theta = -63.4^\circ$

e) Ave speed $= \frac{\text{total distance traveled}}{\text{time}}$

$= \frac{(483+966)}{2.25\,hr} = 644$ km/h

4.22



$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

$x_0 = 0$

$v_{0x} = ?$

$a_x = 0$

$x(t) = v_{0x}t$

$y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

$y_0 = 1.2$

$v_{0y} = 0$

$a_y = -9.8$

$y(t) = 1.2 + \frac{1}{2}(-9.8)t^2$

Strikes ground at

$y = 0 = 1.2 + \frac{1}{2}(-9.8)t^2$

$t = \pm 0.495$ sec

$t = 0.495$ sec

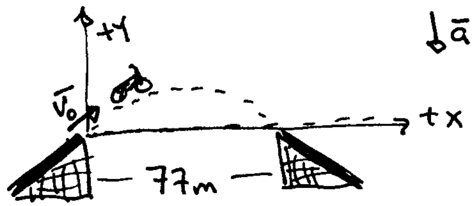
Range is 1.52 m

$x = 1.52 = v_{0x}t$

$1.52 = v_{0x}\,0.495$

$\rightarrow v_{0x} = 3.07$ m/s

4.25



$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

$x_0 = 0$

$v_{0x} = v_0 \cos 12^\circ$

$a_x = 0$

$y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

$y_0 = 0$

$v_{0y} = v_0 \sin 12^\circ$

$a_y = -9.8$ m/s

$x(t) = v_0 \cos 12^\circ\, t$

$y(t) = v_0 \sin 12^\circ\, t + \frac{1}{2}(-9.8)t^2$

we know he came down at $x = 77$ $y = 0$

$77 = v_0 \cos 12^\circ\, t$

$0 = v_0 \sin 12^\circ\, t + \frac{1}{2}(-9.8)t^2$

2 eqns & 2 unknowns

$\frac{77}{v_0 \cos 12^\circ} = t$ → $0 = \frac{v_0 \sin 12^\circ \cdot 77}{v_0 \cos 12^\circ} + \frac{1}{2}(-9.8)t^2$

$0 = \tan 12^\circ \cdot 77 + (-4.9)t^2$

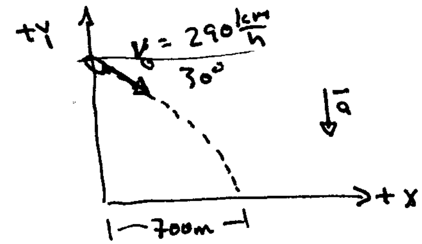
$t = 1.83$ sec

$77 = v_0 \cos 12^\circ\, t$

$77 = v_0 \cos 12^\circ\, (1.83)$

$v_0 = 43.0$ m/s

4.27



$v_0 = 290\frac{km}{h} = 290\frac{1000\,m}{3600\,s}$

$= 80.6$ m/s

$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

$x_0 = 0$

$v_{0x} = 80.6 \cos 30^\circ$

$= 69.8$ m/s

$a_x = 0$

$y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

$y_0 = ?$

$v_{0y} = -80.6 \sin 30^\circ$

$= -40.3$

$a_y = -9.8$

$x(t) = 69.8\, t$

$y(t) = y_0 - 40.3t + \frac{1}{2}(-9.8)t^2$

the radar decoy strikes ground at

$x = 700$ $y = 0$

$700 = 69.8\, t$

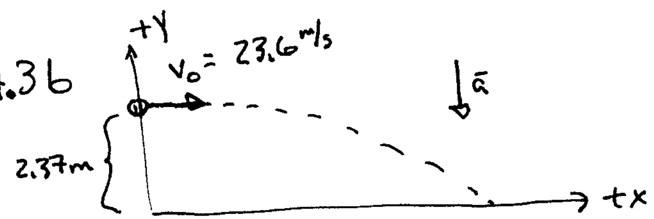
$t = 10.0$ sec

$0 = y_0 - 40.3t + \frac{1}{2}(-9.8)t^2$

$0 = y_0 - 40.3(10) - 4.9(10)^2$

$y_0 = 893$ m

4.36



$x(t) = x_0 + v_{ox}t + \frac{1}{2}a_x t^2$

$x_0 = 0$

$v_{ox} = 23.6$ m/s

$a_x = 0$

$x(t) = 23.6t$

$y(t) = y_0 + v_{oy}t + \frac{1}{2}a_y t^2$

$y_0 = 2.37$ m

$v_{oy} = 0$

$a_y = -9.8$ m/s²

$y(t) = 2.37 + \frac{1}{2}(-9.8)t^2$

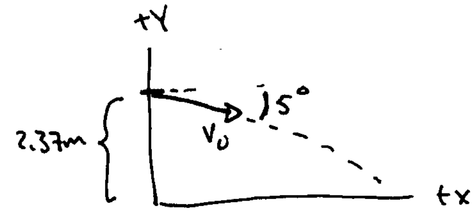
a) How high is the ball when x = 12 m?

$12 = 23.6t$

$t = 0.509$ s → $y = 2.37 + \frac{1}{2}(-9.8)(0.509)^2$

$= 1.10$ m

so the ball clears the top of the net by 0.20 m



$x(t) = x_0 + v_{ox}t + \frac{1}{2}a_x t^2$

$x_0 = 0$

$v_{ox} = 23.6\cos 5°$

$= 23.51$ m/s

$a_x = 0$

$x(t) = 23.5t$

$y(t) = y_0 + v_{oy}t + \frac{1}{2}a_y t^2$

$y_0 = 2.37$ m

$v_{oy} = -23.6\sin 5°$

$= -2.06$ m/s

$a_y = -9.8$ m/s²

$y(t) = 2.37 - 2.06t + \frac{1}{2}(-9.8)t^2$

c) How high is the ball when x = 12 m?

$12 = 23.5t$

$t = 0.511$ → $y = 2.37 - 2.06(0.511) + \frac{1}{2}(-9.8)(0.511)^2$

$= 2.78 \times 10^{-2}$ m

The ball does not clear the net.
The ball is basically on the ground,
height = 2.78 cm.

4.49

$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

$x_0 = 0$

$v_{0x} = 25\cos\theta$

$a_x = 0$

$y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

$y_0 = 0$

$v_{0y} = 25\sin\theta$

$a_y = -9.8\ m/s^2$

$x(t) = 25\cos\theta \cdot t$

$y(t) = 25\sin\theta \cdot t + \frac{1}{2}(-9.8)t^2$

The two extreme cases will just clear the crossbar

$50 = 25\cos\theta \cdot t$

$3.44 = 25\sin\theta \cdot t - 4.9t^2$

OK, the algebra here could be painful.

$\frac{50}{25\cos\theta} = t$

$\frac{2}{\cos\theta} = t$

$\rightarrow 3.44 = 50\frac{\sin\theta}{\cos\theta} - 4.9\left(\frac{2}{\cos\theta}\right)^2$

$3.44 = 50\tan\theta - \frac{19.6}{\cos^2\theta}$

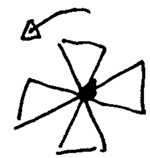
OK, a calculator or spreadsheet is going to need to help me here.

I made a spreadsheet and guessed numbers until I got 2 solutions

$\theta = 31.1°$

$62.8°$

4.58



$1200\ \frac{rev}{min} = \rightarrow$ 1 rev every $\frac{1}{1200}\ min = 8.33\times10^{-4}\ min$ $= 0.05\ sec$

Radius = 0.15 m

a) Circumference of orbit $= 2\pi R$ $= 2\pi(0.15) = 0.942\ m$

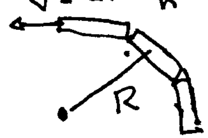
b) Speed of tip ~ $\frac{distance}{time} = \frac{0.942}{0.05} = 18.84\ m/s$

c) Radial acceleration $= \frac{v^2}{R} = \frac{(18.84)^2}{0.942} = 37.7\ m/s^2$

d) Period = time per revolution = 0.05 sec

4.108

$v = 216\ \frac{km}{h} = 216\ \frac{1000\ m}{3600\ s} = 60\ m/s$



a) What is radius for $a_{rad} = 0.050g$ $= 0.050(9.8)$ $= 0.49\ m/s^2$

$a_{rad} = \frac{v^2}{R}$

$0.49 = \frac{(60)^2}{R}$

$R = 7.35\times10^3\ m = 7.35\ km$

b) What speed if $a_{rad} = 0.050g$ and $R = 1000\ m$

$a_{rad} = \frac{v^2}{R}$

$0.49 = \frac{v^2}{1000}$

$v = 22.1\ m/s = 80\ km/hr$

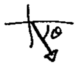
4.6 Electron's position

$\vec{r} = 3t\,\hat{i} - 4t^2\,\hat{j} + 2\,\hat{k}$

a) $\vec{v} = \frac{d\vec{r}}{dt} = 3\hat{i} - 8t\,\hat{j}$

b) at $t=2$ $\quad \vec{v} = 3\hat{i} - 16\hat{j}$

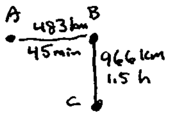
c,d)



$|\vec{v}| = v = \sqrt{3^2+16^2} = 16.3\ m/s$

$\tan\theta = -\frac{16}{3} \rightarrow \theta = -79.4^\circ$

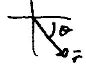
4.8



a) Displacement vector

$\Delta\vec{r} = 483\,\hat{i} - 966\,\hat{j}$ (km)

$|\Delta\vec{r}| = \sqrt{483^2 + 966^2} = 1080\ km$



$\tan\theta = \frac{-966}{483}$

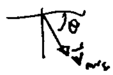
$\rightarrow \theta = -64.1^\circ$

c) $\overrightarrow{\text{Average Velocity}} = \frac{\overrightarrow{\text{net displacement}}}{\text{time}}$

$= \frac{483\,\hat{i} - 966\,\hat{j}}{2.25\,h}$

$= 215\,\hat{i} - 429\,\hat{j}$ (km/hr)

$|\vec{v}_{AVG}| = v_{AVG} = \sqrt{215^2 + 429^2} = 480\ km/hr$

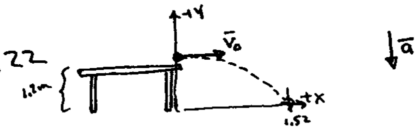


$\tan\theta = \frac{-429}{215} \rightarrow \theta = -63.4^\circ$

e) Ave speed $= \frac{\text{total distance traveled}}{\text{time}}$

$= \frac{(483 + 966)}{2.25\,hr} = 644\ km/h$

4.22



$x(t) = x_o + v_{ox}t + \frac{1}{2}a_x t^2$

$x_o = 0$

$v_{ox} = ?$

$a_x = 0$

$x(t) = v_{ox}t$

$y(t) = y_o + v_{oy}t + \frac{1}{2}a_y t^2$

$y_o = 1.2$

$v_{oy} = 0$

$a_y = -9.8$

$y(t) = 1.2 + \frac{1}{2}(-9.8)t^2$

Strikes ground at

$y = 0 = 1.2 + \frac{1}{2}(-9.8)t^2$

$t = \pm 0.495\ sec$

$t = 0.495\ sec$

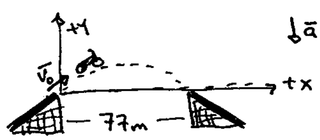
Range is 1.52 m

$x = 1.52 = v_{ox}t$

$1.52 = v_{ox}\ 0.495$

$\rightarrow v_{ox} = 3.07\ m/s$

4.25



$x(t) = x_o + v_{ox}t + \frac{1}{2}a_x t^2$

$x_o = 0$

$v_{ox} = v_o \cos 12^\circ$

$a_x = 0$

$y(t) = y_o + v_{oy}t + \frac{1}{2}a_y t^2$

$y_o = 0$

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$a_y = -9.8\ m/s$

$x(t) = v_o \cos 12^\circ\ t$ $\qquad y(t) = v_o \sin 12^\circ\ t + \frac{1}{2}(-9.8)t^2$

we know he came down at $x = 77$ $y = 0$

$77 = v_o \cos 12^\circ\ t$ $\qquad 0 = v_o \sin 12^\circ\ t + \frac{1}{2}(-9.8)t^2$

2 eqns & 2 unknowns

$\frac{77}{v_o \cos 12^\circ} = t \longrightarrow 0 = \frac{v_o \sin 12^\circ \cdot 77}{v_o \cos 12^\circ} + \frac{1}{2}(-9.8)t^2$

$0 = \tan 12^\circ \cdot 77 + (-4.9)t^2$

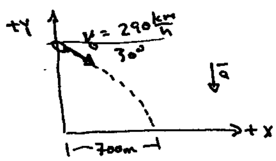
$t = 1.83\ sec$

$77 = v_o \cos 12^\circ\ t$

$77 = v_o \cos 12^\circ\ (1.83)$

$v_o = 43.0\ m/s$

4.27



$v_o = 290\ \frac{km}{h} = 290\ \frac{1000\,m}{3600\,s} = 80.6\ m/s$

$x(t) = x_o + v_{ox}t + \frac{1}{2}a_x t^2$

$x_o = 0$

$v_{ox} = 80.6 \cos 30^\circ = 69.8\ m/s$

$a_x = 0$

$x(t) = 69.8\,t$

$y(t) = y_o + v_{oy}t + \frac{1}{2}a_y t^2$

$y_o = ?$

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$y(t) = y_o - 40.3t + \frac{1}{2}(-9.8)t^2$

the radar decoy strikes ground at $x = 700$ $y = 0$

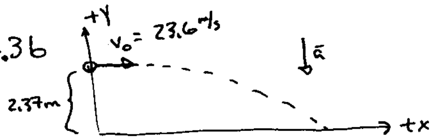
$700 = 69.8\,t$ $\qquad 0 = y_o - 40.3t + \frac{1}{2}(-9.8)t^2$

$t = 10.0\ sec$

$0 = y_o - 40.3(10) - 4.9(10)^2$

$y_o = 893\ m$

4.36



$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

$x_0 = 0$

$v_{0x} = 23.6$ m/s

$a_x = 0$

$x(t) = 23.6t$

$y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

$y_0 = 2.37$ m

$v_{0y} = 0$

$a_y = -9.8$ m/s²

$y(t) = 2.37 + \frac{1}{2}(-9.8)t^2$

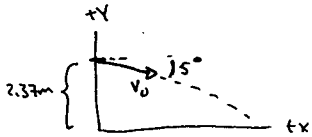
a) How high is the ball when x = 12 m?

$12 = 23.6t$

$t = 0.509$ s $\rightarrow$ $y = 2.37 + \frac{1}{2}(-9.8)(0.509)^2$

$= 1.10$ m

so the ball clears the top of the net by 0.20 m



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$a_y = -9.8$ m/s²

$y(t) = 2.37 - 2.06t + \frac{1}{2}(-9.8)t^2$

c) How high is the ball when x = 12 m?

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$= 2.78 \times 10^{-2}$ m

The ball does not clear the net.
The ball is basically on the ground, height = 2.78 cm.

4.49

$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

$x_0 = 0$

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$a_x = 0$

$x(t) = 25\cos\theta \cdot t$

$y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

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The two extreme cases will just clear the crossbar

$50 = 25\cos\theta \cdot t$ $\quad$ $3.44 = 25\sin\theta \cdot t - 4.9t^2$

ok, the algebra here could be painful.

$\frac{50}{25\cos\theta} = t$ $\rightarrow$ $3.44 = 50\frac{\sin\theta}{\cos\theta} - 4.9\left(\frac{2}{\cos\theta}\right)^2$

$\frac{2}{\cos\theta} = t$

$3.44 = 50\tan\theta - \frac{19.6}{\cos^2\theta}$

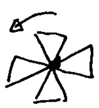
ok, a calculator or spreadsheet is going to need to help me here.

I made a spreadsheet and guessed numbers until I got 2 solutions

$\theta = 31.1°$

$62.8°$

4.58



$1200\frac{rev}{min} =$ $\rightarrow$ 1 rev every $\frac{1}{1200}$ min $= 8.33 \times 10^{-4}$ m

$= 0.05$ sec

Radius = 0.15 m

a) Circumference of orbit $= 2\pi R$

$= 2\pi(0.15) = 0.942$ m

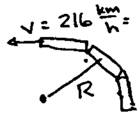
b) Speed of tip ~ $\frac{\text{distance}}{\text{time}} = \frac{0.942}{0.05} = 18.84$ m/s

c) Radial acceleration $= \frac{v^2}{R} = \frac{(18.84)^2}{0.942} = 37.7$ m/s²

d) Period = time per revolution = 0.05 sec

4.108

$v = 216\frac{km}{h} = 216\frac{1000m}{3600s} = 60$ m/s



a) what is radius for $a_{rad} = 0.050g$

$= 0.050(9.8)$

$= 0.49$ m/s²

$a_{rad} = \frac{v^2}{R}$

$0.49 = \frac{(60)^2}{R}$

$R = 7.35 \times 10^3$ m $= 7.35$ km

b) What speed if $a_{rad} = 0.050g$ and $R = 1000$ m

$a_{rad} = \frac{v^2}{R}$

$0.49 = \frac{v^2}{1000}$

$v = 22.1$ m/s $= 80$ km/hr