

10.4  ~~$\theta = A + Bt^3 + Ct^3$~~

$$\theta = 2 + 4t^2 + 2t^3$$

Then $\omega = \frac{d\theta}{dt} = 8t + 6t^2$
 $\alpha = \frac{d\omega}{dt} = 8 + 12t$

a) θ at $t=0$

$$\theta = 2 + 4 \cdot 0^2 + 2 \cdot 0^3 = 2 \text{ rad}$$

b) ω at $t=0$

$$\omega = 8 \cdot 0 + 6 \cdot 0^2 = 0 \text{ rad/s}$$

c) ω at $t=4$

$$\omega = 8 \cdot 4 + 6 \cdot 4^2 = 128 \text{ rad/s}$$

d) α at $t=2$

$$\alpha = 8 + 12 \cdot 2 = 32 \text{ rad/s}^2$$

e) Is $\alpha = \text{constant?}$ Duh no. It has time dependance $\alpha = 8 + 12t$

10.8

 $\alpha = 6t^4 - \frac{1}{4}t^2$

At $t=0 \omega = +2 \text{ rad/s}$ and $\theta = +1.0 \text{ rad}$

Get $\omega(t)$:
 $\omega = \int \omega dt = \int (6t^4 - \frac{1}{4}t^2) dt$
 $= \frac{6}{5}t^5 - \frac{1}{12}t^3 + \text{const}$

then
 $2 = \frac{6}{5}(0)^5 - \frac{1}{12}(0)^3 + \text{const}$
 $\rightarrow \text{const} = 2$

so that

$$\omega(t) = \frac{6}{5}t^5 - \frac{1}{12}t^3 + 2$$

Get $\theta(t)$:

$$\theta = \int \omega dt = \int \left(\frac{6}{5}t^5 - \frac{1}{12}t^3 + 2 \right) dt$$
 $= \frac{6}{30}t^6 - \frac{1}{48}t^4 + 2t + \text{const}$

then
 $1 = \frac{6}{30}0^6 - \frac{1}{48}0^4 + 2 \cdot 0 + \text{const}$
 $\rightarrow \text{const} = 1$

so that
 $\theta(t) = \frac{6}{30}t^6 - \frac{1}{48}t^4 + 2t + 1$

10.10  $\omega_0 = 0$
 const ang accel
 In 5 sec, rotated 25 rad

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\theta = \frac{1}{2}\alpha t^2$$

a) Avg Accel

$$25 = \frac{1}{2}\alpha 5^2$$

$$\alpha = 2 \text{ rad/s}^2$$

b) Ave avg velocity

$$\omega_{\text{ave}} = \frac{\Delta\theta}{\Delta t} = \frac{25 \text{ rad}}{5 \text{ sec}} = 5 \text{ rad/s}$$

c) Avg velocity at 5 sec

$$\theta = \frac{1}{2}\alpha t^2$$

$$\omega = \frac{d\theta}{dt} = \alpha t = 2(5) = 10 \text{ rad/sec}$$

or

$$\omega(t) = \omega_0 + \alpha t = \alpha t = 2(5) = 10 \text{ rad/sec}$$

d) What additional angle b/w $t=5 - 10 \text{ sec}$

$$\text{at } 10 \text{ sec } \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \rightarrow \frac{1}{2}\alpha t^2$$

$$\frac{1}{2} \cdot 2 \cdot 10^2 = 100 \text{ rad}$$

at 5 sec $\theta = 25$

so $\Delta\theta = 100 - 25 = 75 \text{ rad}$

10.11

 $\omega_0 = 120 \text{ rad/s}$
 $\alpha = -4 \text{ rad/s}^2$

$$\omega(t) = \omega_0 + \alpha t \quad \theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\theta_0 = 0$$

$$\omega_0 = 120$$

$$\alpha = -4$$

$$\omega(t) = 120t + \frac{1}{2}(-4)t^2$$

a) steps when $\omega = 0$

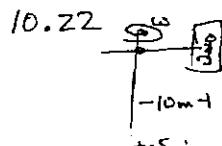
$$0 = 120 - 4t$$

$$t = 30 \text{ sec}$$

b) Angle

$$\theta = 120(30) + \frac{1}{2}(-4)(30)^2$$

$$= 1800 \text{ rad}$$



$$\theta = 0.3t^2$$

When $t=5$:

a) Avg Velocity

$$\omega = \frac{d\theta}{dt} = 0.6t$$

$$= 0.6(5) = 3 \text{ rad/s}$$

b) Linear Velocity

$$v = R\omega = (10)(3) = 30 \text{ m/s}$$

c) Tangential accel

$$\alpha = \frac{d\omega}{dt} = 0.6 \text{ rad/s}^2$$

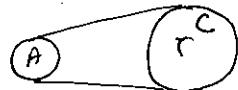
so that

$$a_{\text{tangential}} = R\alpha = 10(0.6) = 6 \text{ m/s}^2$$

d) Radial Accel

$$a_{\text{rad}} = \frac{v^2}{R} = \frac{(30)^2}{10} = 90 \text{ m/s}^2$$

10.28



$$r_A = 10 \text{ cm}$$

$$r_C = 25 \text{ cm}$$

Angular speed of A increases at $\alpha = 1.6 \text{ rad/s}^2$
starts from rest.

The angular speed of pulley A is

$$\omega_A(t) = \frac{1}{2}\alpha t + \omega_0$$

$$= 1.6t$$

The linear speed of the belt is

$$V_{\text{belt}} = R_A \omega_A = (10 \text{ cm})(1.6t)$$

$$= 16t$$

The angular speed of pulley C is

$$\omega_C = \frac{V_{\text{belt}}}{R_C} = \frac{16t}{25}$$

How long does it take for pulley C
to reach $100 \text{ rev/min} = 100 \frac{2\pi \text{ rad}}{60 \text{ sec}}$

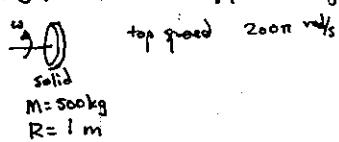
$$= 10.47 \text{ rad/s}$$

Then

$$\omega_C = 10.47 = \frac{16t}{25}$$

$$\rightarrow t = 16.4 \text{ sec}$$

10.39 Stored energy in a flywheel



a) Max KE in flywheel.

$$\text{rot KE} = \frac{1}{2} I \omega^2$$

$$\text{disk } I = \frac{1}{2} MR^2 = \frac{1}{2} 500(1)^2$$

$$= 250 \text{ kgm}^2$$

$$\text{rot KE} = \frac{1}{2} 250 (200\pi)^2 = 49.3 \text{ MegaJoules}$$

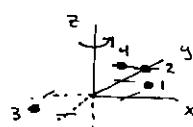
b) Suppose uses energy at 8 kW, how long can it operate?

$$\text{Power} = \frac{\text{Energy}}{\text{time}}$$

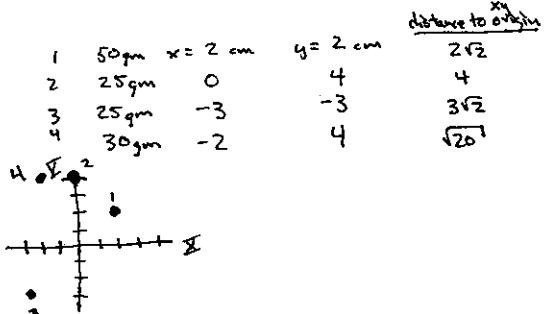
$$8 \times 10^3 = \frac{49.3 \times 10^6}{\text{time}}$$

$$\text{time} = 6168 \text{ sec}$$

$$= 103 \text{ min}$$



10.42



a) Spin about the x-axis.

in the formula $I = \sum m_i r_i^2$ - the "r_i" is y-distance

$$I_x = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

$$= 50(2)^2 + 25(4)^2 + 25(-3)^2 + 30(-4)^2$$

$$= 1305 \text{ g cm}^2$$

b) Spin about the y-axis.

in the formula $I = \sum m_i r_i^2$ - the "r_i" is |x-dist|

$$I_y = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

$$= 50(2)^2 + 25(0)^2 + 25(3)^2 + 30(2)^2$$

$$= 545 \text{ g cm}^2$$

c) Spin about the z axis

in the formula $I = \sum m_i r_i^2$ - the "r_i" is distance to origin

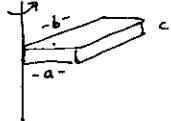
$$I_z = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

$$= 50(2\sqrt{2})^2 + 25(4)^2 + 25(3\sqrt{2})^2 + 30(6\sqrt{2})^2$$

$$= 1850 \text{ g cm}^2$$

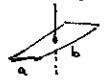
d) $I_x + I_y = I_z$

10.43



$$\begin{aligned}M &= 0.172 \text{ kg} \\a &= 3.5 \text{ cm} \\b &= 8.4 \text{ cm} \\c &= 1.4 \text{ cm}\end{aligned}$$

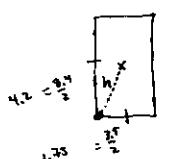
Table 10-2 gives moment of inertia of rectangular slab through center



$$\begin{aligned}I &= \frac{1}{12} M(a^2 + b^2) \\&= \frac{1}{12} 0.172 (3.5^2 + 8.4^2) \\&= 1.187 \text{ g} \cdot \text{cm}^2\end{aligned}$$

Use the parallel axis theorem to shift the moment of inertia from cm to the corner

$$I = I_{cm} + Mh^2$$



$$\begin{aligned}4.2 &= \frac{b}{2} \\1.75 &= \frac{b}{2}\end{aligned}$$

$$\begin{aligned}I &= 1.187 + 0.172(4.55)^2 \\I &= 4.75 \text{ g} \cdot \text{cm}^2\end{aligned}$$

$$\begin{aligned}h^2 &= (1.75)^2 + (4.2)^2 \\h &= 4.55 \text{ cm}\end{aligned}$$

10.4

$$\theta = 2 + 4t^2 + 2t^3$$

then $\omega = \frac{d\theta}{dt} = 8t + 6t^2$
 $\alpha = \frac{d\omega}{dt} = 8 + 12t$

a) θ at $t=0$
 $\theta = 2 + 4 \cdot 0^2 + 2 \cdot 0^3 = 2 \text{ rad}$

b) ω at $t=0$
 $\omega = 8 \cdot 0 + 6 \cdot 0^2 = 0 \text{ rad/s}$

c) ω at $t=4$
 $\omega = 8 \cdot 4 + 6 \cdot 4^2 = 128 \text{ rad/s}$

d) α at $t=2$
 $\alpha = 8 + 12 \cdot 2 = 32 \text{ rad/s}^2$

e) Is α constant? Duh no. It has time dependence $\alpha = 8 + 12t$

10.8

$$\alpha = 6t^4 - \frac{4}{3}t^2$$

$$\bullet$$

At $t=0$ $\omega = +2 \text{ rad/s}$ and $\theta = +1.0 \text{ rad}$

Get $\omega(t)$:
 $\omega = \int \alpha dt = \int (6t^4 - \frac{4}{3}t^2) dt$
 $= \frac{6}{5}t^5 - \frac{4}{3}t^3 + \text{const}$

then $\omega = \frac{6}{5}(0)^5 - \frac{4}{3}(0)^3 + \text{const}$

$$\rightarrow \text{const} = 2$$

so that
 $\omega(t) = \frac{6}{5}t^5 - \frac{4}{3}t^3 + 2$

Get $\theta(t)$:

$$\begin{aligned} \theta &= \int \omega dt = \int \left(\frac{6}{5}t^5 - \frac{4}{3}t^3 + 2 \right) dt \\ &= \frac{6}{5 \cdot 6}t^6 - \frac{4}{3 \cdot 4}t^4 + 2t + \text{const} \\ \text{then } I &= \frac{6}{30}t^6 - \frac{4}{12}t^4 + 2t + \text{const} \\ \rightarrow \text{const} &= 1 \end{aligned}$$

so that
 $\theta(t) = \frac{6}{30}t^6 - \frac{4}{12}t^4 + 2t + 1$

10.10  $\omega_0 = 0$
const ang accel
In 5 sec, rotated 25 rad

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\theta_0 = 0$$

$$\omega_0 = 0$$

$$\alpha = \frac{1}{2} \alpha t^2$$

a) Avg Accel

$$\theta = \frac{1}{2} \alpha t^2$$

$$25 = \frac{1}{2} \alpha 5^2$$

$$\alpha = 2 \text{ rad/sec}^2$$

b) Ave ang velocity

$$\omega_{AVE} = \frac{\Delta \theta}{\Delta t} = \frac{25 \text{ rad}}{5 \text{ sec}} \approx 5 \text{ rad/sec}$$

c) Avg velocity at 5 sec

$$\theta = \frac{1}{2} \alpha t^2$$

$$\omega = \frac{d\theta}{dt} = \alpha t = 2(5) = 10 \text{ rad/sec}$$

or

$$\omega(t) = \omega_0 + \alpha t = \alpha t = 2(5) = 10 \text{ rad/sec}$$

d) What additional angle b/w $t = 5 - 10 \text{ sec}$

$$\text{at } 10 \text{ sec } \theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \rightarrow \frac{1}{2} \alpha t^2$$

$$\frac{1}{2} 2 10^2 = 100 \text{ rad}$$

at 5 sec $\theta = 25$

so $\Delta\theta = 100 - 25 = 75 \text{ rad}$

10.11



$$\omega_0 = 120 \text{ rad/s}$$

$$\alpha = -4 \text{ rad/sec}^2$$

$$\omega(t) = \omega_0 + \alpha t$$

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\theta_0 = 0$$

$$\omega_0 = 120$$

$$\alpha = -4$$

$$\omega(t) = 120 - 4t$$

$$\theta(t) = 120t + \frac{1}{2}(-4)t^2$$

a) steps when $\omega = 0$

$$0 = 120 - 4t$$

$$t = 30 \text{ sec}$$

$$\theta(t) = 120(30) + \frac{1}{2}(-4)(30)^2$$

$$= 1800 \text{ rad}$$

b) Angle ω

$$\theta = \frac{1}{2} \alpha t^2$$

$$\omega = \frac{d\theta}{dt} = 2(5) = 10 \text{ rad/sec}$$

or

$$\omega(t) = \omega_0 + \alpha t = \alpha t = 2(5) = 10 \text{ rad/sec}$$

$$10.22 \quad \begin{array}{c} \omega \\ \text{---} \\ -10\text{m} \end{array} \quad \theta = 0.3t^2$$

when $t=5$:

a) Ang velocity $\omega = \frac{d\theta}{dt} = 0.6t = 0.6(5) = 3 \text{ rad/s}$

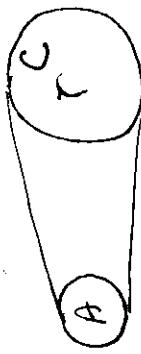
b) Linear velocity $v = R\omega = (10)(3) = 30 \text{ m/s}$

c) Tangential accel $\alpha = \frac{d\omega}{dt} = 0.6 \text{ rad/s}^2$

so that $a_{\text{tangential}} = R\alpha = 10(0.6) = 6 \text{ m/s}^2$

d) Radial Accel $a_{\text{rad}} = \frac{v^2}{R} = \frac{(30)^2}{10} = 90 \text{ m/s}^2$

10.28



$$r_A = 10 \text{ cm} \quad v_C = 25 \text{ cm}$$

Angular speed of A increases at $\alpha = 1.6 \text{ rad/s}^2$
starts from rest.

The angular speed of pulley A is
 $\omega_A(t) = \int_0^t \alpha dt + \omega_0$

$$= 1.6t$$

The linear speed of the belt is
 $v = R_A \omega_A = (10 \text{ cm})[1.6t]$

The angular speed of pulley C is
 $\omega_C = \frac{v_{\text{belt}}}{R_C} = \frac{16t}{25}$

How long does it take for pulley C
to reach 100 rev/min = $100 \frac{\text{rev}}{60 \text{ sec}} = \frac{2\pi}{3} \text{ rad/sec}$
= 10.47 rad/s

then

$$\omega_C = 10.47 = \frac{16t}{25}$$

$$\Rightarrow t = 16.4 \text{ sec}$$

10.39 Stored energy in a flywheel

top speed 200π rev/s



$$M = 500 \text{ kg}$$

$$R = 1 \text{ m}$$

a) Max KE in flywheel.

$$\text{rot KE} = \frac{1}{2} I \omega^2$$

$$\text{disk } I = \frac{1}{2} m R^2 = \frac{1}{2} 500 (1)^2$$

$$= 250 \text{ kgm}^2$$

$$\text{rot KE} = \frac{1}{2} 250 (200\pi)^2 = 49.3 \text{ megajoules}$$

b) Suppose uses energy at 8 kW, how long can it operate?

$$\text{Power} = \frac{\text{Energy}}{\text{time}}$$

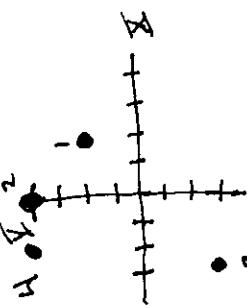
$$8 \times 10^3 = \frac{49.3 \times 10^6}{\text{time}}$$

$$\text{time} = 6168 \text{ sec}$$

$$= 103 \text{ min}$$

10.42

		x = 2 cm	y = 2 cm	$2\sqrt{2}$
1	50 gm	2	4	4
2	25 gm	0	-3	$3\sqrt{2}$
3	25 gm	-3	4	$\sqrt{20}$
4	30 gm	-2		



a) Spin about the x-axis.
in the formula $I = \sum m_i r_i^2$ - the "r" is |y-distance|

$$I_x = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

$$= 50(2)^2 + 25(4)^2 + 25(3)^2 + 30(4)^2$$

$$= 1355 \text{ gcm}^2$$

b) Spin about the y-axis.
in the formula $I = \sum m_i r_i^2$ - the "r" is |x-distance|

$$I_y = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

$$= 50(2)^2 + 25(0)^2 + 25(3)^2 + 30(2)^2$$

$$= 545 \text{ gcm}^2$$

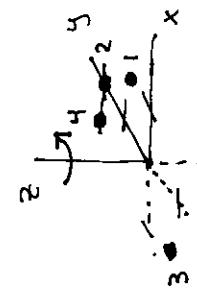
c) Spin about the z-axis - the "r" is distance from the origin

$$I_z = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

$$= 50(2\sqrt{2})^2 + 25(4)^2 + 25(3\sqrt{2})^2 + 30(\sqrt{20})^2$$

$$= 1850 \text{ gcm}^2$$

$$I_x + I_y = I_z$$



10.43

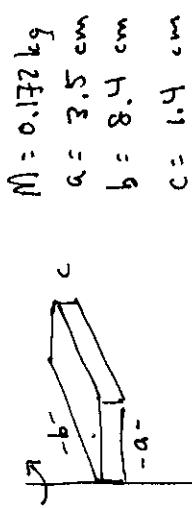
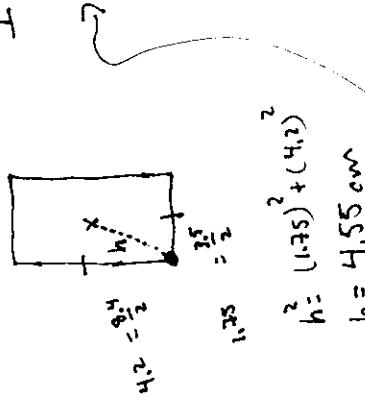


Table 10-2 gives moment of inertia of rectangular slab through center

$$\begin{aligned} I &= \frac{1}{12} M(a^2 + b^2) \\ &= \frac{1}{12} 0.172 (3.5^2 + 8.4^2) \\ &= 1.187 \text{ g.cm}^2 \end{aligned}$$

Use the parallel axis theorem to shift the moment of inertia from center to the corner

$$I = I_{cm} + Mh^2$$



$$\begin{aligned} I &= 1.187 + 0.172(5.19)^2 \\ I &= 4.75 \text{ g.cm}^2 \end{aligned}$$

$$\begin{aligned} h^2 &= (1.75)^2 + (4.2)^2 \\ h &= 4.55 \text{ cm} \end{aligned}$$