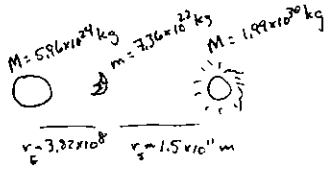
13.4



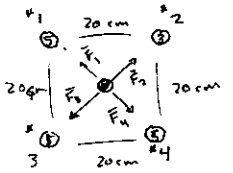
$$\frac{F_{\text{on moon by Sun}}}{F_{\text{on moon by Earth}}} = \frac{G\frac{m M_s}{r_s^2}}{G\frac{m M_E}{r_E^2}}$$

$$= \frac{M_s}{M_E}\left(\frac{r_E}{r_s}\right)^2$$

$$= \frac{1.99\times10^{30}}{5.96\times10^{24}}\left(\frac{3.82\times10^8}{1.5\times10^{11}}\right)^2$$

$$= 2.17$$

13.6



$m_1 = 5\times10^{-3}$ kg
$m_2 = 3\times10^{-3}$ kg
$m_3 = 1\times10^{-3}$ kg
$m_4 = 5\times10^{-3}$ kg
$m_5 = 2.5\times10^{-3}$ kg

distance to center from a corner $r = 10\sqrt{2}$ cm $= 14.14$ cm

First, notice that forces $\vec{F}_1$ and $\vec{F}_4$ will cancel because they have same magnitude and opposite direction.

$$|\vec{F}_2| = F_2 = G\frac{m_5 M_2}{r^2} = (6.67\text{-}11)\frac{(2.5\times10^{-3})(3\times10^{-3})}{(0.1414)^2}$$

$$= 2.5\times10^{-14} \text{ Newtons}$$

$$|\vec{F}_3| = F_3 = G\frac{m_5 M_3}{r^2} = (6.67\times10^{-11})\frac{(2.5\times10^{-3})(1\times10^{-3})}{(0.1414)^2}$$

$$= 8.34\times10^{-15} \text{ Newtons}$$

$$= 0.834\times10^{-14} \text{ Newtons}$$

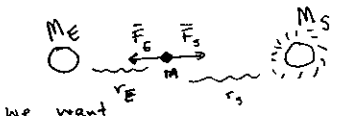
then

$$\vec{F}_2 = F_2\cos 45^\circ\,\hat{i} + F_2\sin 45^\circ\,\hat{j}$$
$$\vec{F}_3 = -F_3\cos 45^\circ\,\hat{i} + -F_3\sin 45^\circ\,\hat{j}$$

$$\vec{F}_{Tot} = (F_2 - F_3)\cos 45^\circ\,\hat{i} + (F_2 - F_3)\sin 45^\circ\,\hat{j}$$

$$= 1.18\times10^{-14}\,\hat{i} + 1.18\times10^{-14}\,\hat{j} \text{ Newtons}$$

13.9



$M_S = 1.99\times10^{30}$ kg
$M_E = 5.96\times10^{24}$ kg

we want

$$F_{Earth} = F_{Sun}$$

$$G\frac{m M_E}{r_E^2} = G\frac{m M_S}{r_S^2}$$

$$\left(\frac{r_S}{r_E}\right)^2 = \frac{M_S}{M_E} = \frac{1.99\times10^{30}}{5.96\times10^{24}}$$

$$\frac{r_S}{r_E} = 577.8$$

Also know

$$r_E + r_S = 1.5\times10^{11}\text{ m}$$ Earth-Sun distance

so

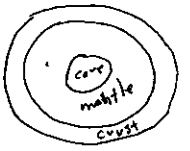
$$r_E + 577.8\, r_E = 1.5\times10^{11}$$

$$578.8\, r_E = 1.5\times10^{11}$$

$$r_E = 2.6\times10^8 \text{ m}$$

13.27

$$|F| = F = G\frac{mM}{r^2}$$ what we call g.



$M_{core} = 1.93\times10^{24}$ kg $r_{core} = 3490$ km
$M_{mantle} = 4.01\times10^{24}$ kg $r_{mantle\ outer} = 6345$ km
$M_{crust} = 3.94\times10^{22}$ kg $r_{crust\ outer} = 6370$ km

5.94

a) g at surface of crust

$$g = G\frac{M_{\text{whole earth}}}{r^2_{\text{crust outer}}} = (6.67\times10^{-11})\frac{5.94\times10^{24}}{(6370\times10^3)^2}$$

$$= 9.76 \text{ m/s}^2$$

b) g at top of mantle

$$g = G\frac{(M_{core} + M_{mantle})}{r^2_{\text{mantle outer}}} = 9.84 \text{ m/s}^2$$

c) Suppose Earth were uniform density. Calc g at top of mantle. Estimate mass ~~th~~ mantle and below by

$$M = 5.96\times10^{24}\,\frac{\frac{4}{3}\pi r^3_{mantle}}{\frac{4}{3}\pi r^3_{crust}}$$

$$= 5.89\times10^{24}$$

then

$$g = G\frac{M}{r^2} = (6.67\times10^{-11})\frac{5.89\times10^{24}}{(6345\times10^3)^2}$$

$$= 9.758 \text{ m/s}^2$$

13.39

Asteroid

$M = ?$

$r = 500\ km = 500 \times 10^3\ m$

$g_{surf} = 3.0\ m/s$

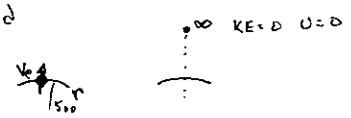
get mass from

$$g = G\frac{M}{r^2}$$

$$3.0 = (6.67\times10^{-11})\frac{M}{(500\times10^3)^2}$$

$$M = 1.124\times10^{22}\ kg$$

a) Escape Speed



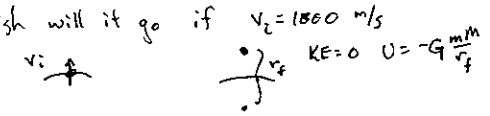
Tot Init E = Tot Final E

$$\tfrac{1}{2}mv_e^2 + -G\frac{mM}{r} = 0^{KE} + 0^{PE}$$

$$v_e^2 = \frac{2GM}{r} = \frac{2(6.67\times10^{-11})(1.124\times10^{22})}{(500\times10^3)}$$

$$v_e = 1.73\times10^3\ m/s = 1.73\ km/s$$

b) How high will it go if $v_i = 1000\ m/s$



Tot Init E = Tot Final E

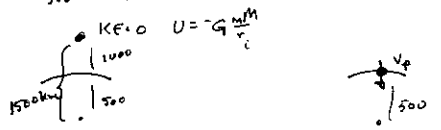
$$\tfrac{1}{2}\cancel{m}v_i^2 + -G\frac{\cancel{m}M}{r} = 0 + -G\frac{\cancel{m}M}{r_f}$$

$$\tfrac{1}{2}(1000)^2 - (6.67\times10^{-11})\frac{1.124\times10^{22}}{500\times10^3} = -(6.67\times10^{-11})\frac{1.124\times10^{22}}{r_f}$$

$$r_f = 7.5\times10^5\ m = 750\ km$$

or 250 km above surface

c) What speed will it hit surface if dropped from 1000 km above surface?



Tot Init E = Tot Final E

$$0 + -G\frac{\cancel{m}M}{r_i} = \tfrac{1}{2}\cancel{m}v_f^2 + -G\frac{\cancel{m}M}{r}$$

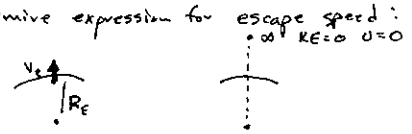
$$-(6.67\times10^{-11})\frac{1.124\times10^{22}}{1500\times10^3} = \tfrac{1}{2}v_f^2 - (6.67\times10^{-11})\frac{1.124\times10^{22}}{500\times10^3}$$

$$v_f = 1.41\times10^3\ m/s$$

$$= 1.41\ km/s$$

13.40 Earth escape speed = 11.2 km/s

To determine expression for escape speed:



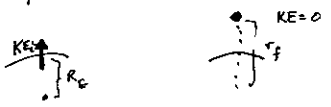
Tot Init E = Tot Fin E

$$\underbrace{\tfrac{1}{2}mv_e^2}_{KE} + -G\frac{mM}{R_E} = 0 + 0$$

$$KE_{escape} = G\frac{mM}{R_E}$$

$$\tfrac{1}{2}mv_e^2 = G\frac{mM}{R_E}$$

Now ~~after~~ we've got these formula, set the actual problem up.



Tot Init E = Total Final E

$$KE + -G\frac{mM}{R_E} = 0 + -G\frac{mM}{r_f}$$

a) $v_i = \frac{1}{2}$ escape speed

Note that this implies $KE_i = \frac{1}{4}KE_{escape}$

so

$$\tfrac{1}{4}KE_{escape} - G\frac{mM}{R_E} = -G\frac{mM}{r_f}$$

$$\tfrac{1}{4}G\frac{mM}{R_E} - G\frac{mM}{R_E} = -G\frac{mM}{r_f}$$

$$-\tfrac{3}{4}G\frac{mM}{R_E} = -G\frac{mM}{r_f}$$

$$r_f = \tfrac{4}{3}R_E$$

b) $KE_i = \frac{1}{2}KE_{escape}$

so

$$\tfrac{1}{2}KE_{escape} - G\frac{mM}{R_E} = -G\frac{mM}{r_f}$$

$$\tfrac{1}{2}G\frac{mM}{R_E} - G\frac{mM}{R_E} = -G\frac{mM}{r_f}$$

$$-\tfrac{1}{2}G\frac{mM}{R_E} = -G\frac{mM}{r_f}$$

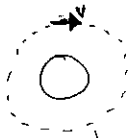
$$r_f = 2R_E$$

c) tricky wording.

Because Mechanical Energy = KE + Pot'l E

the answer is <u>zero</u>.

13.43



$M = 5.96 \times 10^{24}$ kg

$r = 6370\text{ km} + 160\text{ km}$
$= 6530$ km

Do fbd for the circular motion

$$m a_{rad} = \sum F_{radial}$$

$$\cancel{m}\frac{v^2}{r} = G\frac{\cancel{m}M}{r^2}$$

$$v^2 = G\frac{M}{r} = (6.67\times10^{-11})\frac{5.96\times10^{24}}{6530\times10^3}$$

$$v = 7.80\times10^3 \text{ m/s} = 7.8 \text{ km/s}$$

Period

$$v = \frac{2\pi r}{T}$$

$$7.80\times10^3 = \frac{2\pi(6530\times10^3)}{T}$$

$$T = 5.26\times10^3 \text{ sec} = 1.46 \text{ hrs}$$

13.45

$r = 9.4\times10^6$ m

$T = 7h\,39m = 2.754\times10^4$ sec

We could just start with Kepler's 2-3 Rule

$$T^2 = \frac{4\pi^2}{GM}r^3$$

$$(2.754\times10^4)^2 = \frac{4\pi^2}{(6.67\times10^{-11})M}(9.4\times10^6)^3$$

$$M = 6.48\times10^{23} \text{ kg}$$

13.47

$r = 2.2\times10^{20}$ m

$T = 2.5\times10^8 y = 7.85\times10^{15}$ s

$$T^2 = \frac{4\pi^2}{GM}r^3$$

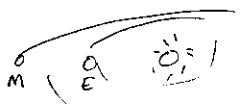
$$(7.85\times10^{15})^2 = \frac{4\pi^2}{(6.67\times10^{-11})M}(2.2\times10^{20})^3$$

$$M = 1.02\times10^{41} \text{ kg}$$

If each star is $\sim 2\times10^{30}$ kg, then there are

$$\frac{1.02\times10^{41}}{2\times10^{30}} \sim 5.1\times10^{10} \text{ stars}$$

13.48



$$T^2 = \frac{4\pi^2}{GM_{sun}}r^3$$

Form a ratio

$$\frac{T_m^2 = \frac{4\pi^2}{GM_{sun}}r_m^3}{T_E^2 = \frac{4\pi}{GM_{sun}}r_E^3}$$

$$\left(\frac{T_m}{T_E}\right)^2 = \left(\frac{r_m}{r_E}\right)^3 = (1.52)^3$$

$$\frac{T_m}{T_E} = 1.87$$

$$T_m = 1.87\ T_E$$

$$T_m = 1.87 \text{ years}$$

Value in back of book is the same ~1.88 years

13.56

take $r \sim 100$ km

$T = 27h = 9.72\times10^4$ s

a) Use $T^2 = \frac{4\pi^2}{GM}r^3$

$$(9.72\times10^4)^2 = \frac{4\pi^2}{(6.67\times10^{-11})M}(100\times10^3)^3$$

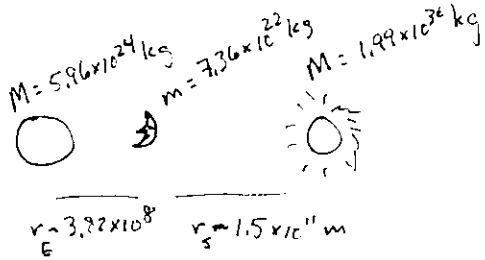
$$M = 6.26\times10^{16} \text{ kg}$$

b) Volume $= 14100\,(km)^3 = 1.41\times10^{13}\ m^3$

$$\text{density} = \frac{\text{mass}}{\text{vol}} = \frac{6.26\times10^{16}}{1.41\times10^{13}} = 4.44\times10^3 \text{ kg/m}^3$$

$$= 4.44 \text{ g/cm}^3$$

13.4



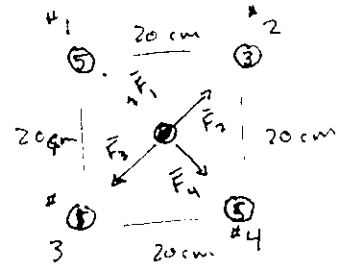
$$\frac{F_{\text{on moon by Sun}}}{F_{\text{on moon by Earth}}} = \frac{G\frac{m M_S}{r_S^2}}{G\frac{m M_E}{r_E^2}}$$

$$= \frac{M_S}{M_E}\left(\frac{r_E}{r_S}\right)^2$$

$$= \frac{1.99\times10^{30}}{5.96\times10^{24}}\left(\frac{3.82\times10^{8}}{1.5\times10^{11}}\right)^2$$

$$= 2.17$$

13.6



$m_1 = 5\times10^{-3}$ kg
$m_2 = 3\times10^{-3}$ kg
$m_3 = 1\times10^{-3}$ kg
$m_4 = 5\times10^{-3}$ kg
$m_5 = 2.5\times10^{-3}$ kg

distance to center from a corner $r = 10\sqrt{2}$ cm $= 14.14$ cm

First, notice that forces $\bar{F}_1$ and $\bar{F}_4$ will cancel because they have same magnitude and opposite direction.

$$|\bar{F}_2| = F_2 = G\frac{m_5 M_2}{r^2} = (6.67\text{-}11)\frac{(2.5\times10^{-3})(3\times10^{-3})}{(0.1414)^2}$$

$$= 2.5\times10^{-14} \text{ Newtons}$$

$$|\bar{F}_3| = F_3 = G\frac{m_5 M_3}{r^2} = (6.67\times10^{-11})\frac{(2.5\times10^{-3})(1\times10^{-3})}{(0.1414)^2}$$

$$= 8.34\times10^{-15} \text{ Newtons}$$

$$= 0.834\times10^{-14} \text{ Newtons}$$

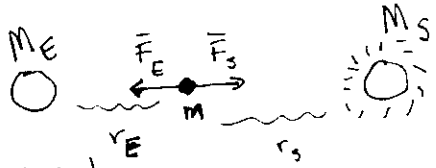
then

$$\vec{F}_2 = F_2\cos 45^\circ\,\hat{i} + F_2\sin 45^\circ\,\hat{j}$$
$$\vec{F}_3 = -F_3\cos 45^\circ\,\hat{i} + -F_3\sin 45^\circ\,\hat{j}$$

$$\bar{F}_{Tot} = (F_2 - F_3)\cos 45^\circ\,\hat{i} + (F_2 - F_3)\sin 45^\circ\,\hat{j}$$

$$= 1.18\times10^{-14}\,\hat{i} + 1.18\times10^{-14}\,\hat{j} \text{ Newtons}$$

13.9



$M_S = 1.99 \times 10^{30}$ kg

$M_E = 5.96 \times 10^{24}$ kg

we want

$$F_{Earth} = F_{Sun}$$

$$G \frac{m M_E}{r_E^2} = G \frac{m M_S}{r_S^2}$$

$$\left(\frac{r_S}{r_E}\right)^2 = \frac{M_S}{M_E} = \frac{1.99 \times 10^{30}}{5.96 \times 10^{24}}$$

$$\frac{r_S}{r_E} = 577.8$$

Also know

$$r_E + r_S = 1.5 \times 10^{11} \text{ m}$$ Earth-Sun distance

so

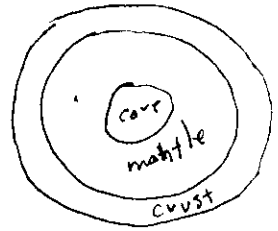
$$r_E + 577.8\, r_E = 1.5 \times 10^{11}$$

$$578.8\, r_E = 1.5 \times 10^{11}$$

$$r_E = 2.6 \times 10^{8} \text{ m}$$

13.27

$$|F| = F = G \frac{mM}{r^2}$$ what we call g.



$M_{core} = 1.93 \times 10^{24}$ kg $\quad r_{core} = 3496$ km

$M_{mantle} = 4.01 \times 10^{24}$ kg $\quad r_{mantle\ outer} = 6345$ km

$M_{crust} = \frac{3.94 \times 10^{22}}{5.94}$ kg $\quad r_{crust\ outer} = 6370$ km

a) g at surface of crust

$$g = G \frac{M_{whole\ earth}}{r^2_{crust\ outer}} = (6.67 \times 10^{-11}) \frac{5.94 \times 10^{24}}{(6370 \times 10^3)^2}$$

$$= 9.76 \text{ m/s}^2$$

b) g at top of mantle

$$g = G \frac{(M_{core} + M_{mantle})}{r^2_{mantle\ outer}} = 9.84 \text{ m/s}^2$$

c) Suppose Earth were uniform density. Calc g at top of mantle. Estimate mass in mantle and below by

$$M = 5.96 \times 10^{24} \frac{\frac{4}{3}\pi r^3_{mantle}}{\frac{4}{3}\pi r^3_{crust}}$$

$$= 5.89 \times 10^{24}$$

then

$$g = G \frac{M}{r^2} = (6.67 \times 10^{-11}) \frac{5.89 \times 10^{24}}{(6345 \times 10^3)^2}$$

$$= 9.758 \text{ m/s}^2$$

13.39

Asteroid

$M = ?$

$r = 500\ km = 500 \times 10^3\ m$

$g_{surf} = 3.0\ m/s$

get mass from

$$g = G\frac{M}{r^2}$$

$$3.0 = (6.67\times10^{-11})\frac{M}{(500\times10^3)^2}$$

$$M = 1.124\times10^{22}\ kg$$

a) Escape Speed

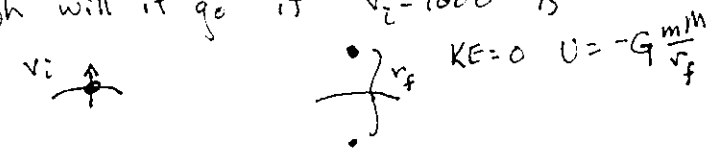
Tot Init E = Tot Final E

$$\frac{1}{2}mv_e^2 + -G\frac{mM}{r} = 0^{KE} + 0^{PE}$$

$$v_e^2 = \frac{2GM}{r} = \frac{2(6.67\times10^{-11})(1.124\times10^{22})}{(500\times10^3)}$$

$$v_e = 1.73\times10^3\ m/s = 1.73\ km/s$$

b) How high will it go if $v_i = 1000\ m/s$



Tot Init E = Tot Final E

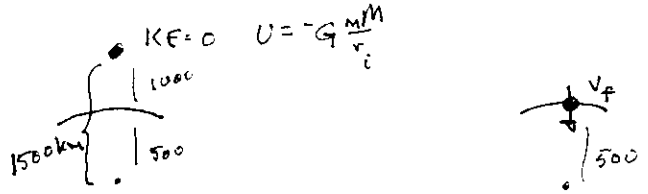
$$\frac{1}{2}mv_i^2 + -G\frac{mM}{r} = 0 + -G\frac{mM}{r_f}$$

$$\frac{1}{2}(1000)^2 - (6.67\times10^{-11})\frac{1.124\times10^{22}}{500\times10^3} = -(6.67\times10^{-11})\frac{1.124\times10^{22}}{r_f}$$

$$r_f = 7.5\times10^5\ m = 750\ km$$

or 250 km above surface

c) What speed will it hit surface if dropped from 1000 km above surface?



Tot Init E = Tot Final E

$$0 + -G\frac{mM}{r_i} = \frac{1}{2}mv_f^2 + -G\frac{mM}{r}$$

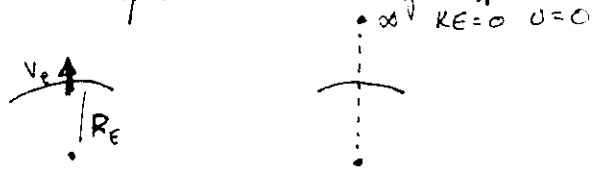
$$-(6.67\times10^{-11})\frac{1.124\times10^{22}}{1500\times10^3} = \frac{1}{2}v_f^2 - (6.67\times10^{-11})\frac{1.124\times10^{22}}{500\times10^3}$$

$$v_f = 1.41\times10^3\ m/s$$

$$= 1.41\ km/s$$

13.40 Earth escape speed = 11.2 km/s

To determine expression for escape speed:



Tot Init E = Tot Fin E

$$\underbrace{\tfrac{1}{2}mv_e^2}_{KE} + -G\frac{mM}{R_E} = 0 + 0$$

$$KE_{escape} = G\frac{mM}{R_E}$$

$$\tfrac{1}{2}mv_e^2 = G\frac{mM}{R_E}$$

Now ~~after~~ we've got these formula, set the actual problem up.

Tot Init E = Total Final E

$$KE + -G\frac{mM}{R_E} = 0 + -G\frac{mM}{r_f}$$

a) $v_i = \frac{1}{2}$ escape speed

Note that this implies $KE_i = \frac{1}{4}KE_{escape}$

so

$$\tfrac{1}{4}KE_{escape} - G\frac{mM}{R_E} = -G\frac{mM}{r_f}$$

$$\tfrac{1}{4}G\frac{mM}{R_E} - G\frac{mM}{R_E} = -G\frac{mM}{r_f}$$

$$-\tfrac{3}{4}G\frac{mM}{R_E} = -G\frac{mM}{r_f}$$

$$r_f = \tfrac{4}{3}R_E$$

b) $KE_i = \frac{1}{2}KE_{escape}$

so

$$\tfrac{1}{2}KE_{escape} - G\frac{mM}{R_E} = -G\frac{mM}{r_f}$$

$$\tfrac{1}{2}G\frac{mM}{R_E} - G\frac{mM}{R_E} = -G\frac{mM}{r_f}$$

$$-\tfrac{1}{2}G\frac{mM}{R_E} = -G\frac{mM}{r_f}$$

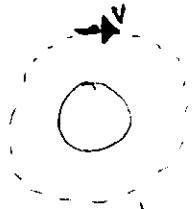
$$r_f = 2R_E$$

c) tricky wording.

Because Mechanical Energy = KE + Pot'l E

the answer is <u>zero</u>.

13.43



$M = 5.96 \times 10^{24}$ kg

$r = 6370\text{ km} + 160\text{ km}$
$= 6530$ km

Do fbd for the circular motion

$$m a_{rad} = \sum F_{radial}$$

$$\cancel{m}\frac{v^2}{r} = G\frac{\cancel{m}M}{r^2}$$

$$v^2 = G\frac{M}{r} = (6.67\times10^{-11})\frac{5.96\times10^{24}}{6530\times10^3}$$

$$v = 7.80\times10^3 \text{ m/s} = 7.8 \text{ km/s}$$

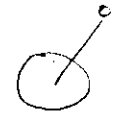
Period

$$v = \frac{2\pi r}{T}$$

$$7.80\times10^3 = \frac{2\pi(6530\times10^3)}{T}$$

$$T = 5.26\times10^3 \text{ sec} = 1.46 \text{ hrs}$$

13.45



$r = 9.4\times10^6$ m

$T = 7\text{h } 39\text{m} = 2.754\times10^4$ sec

We could just start with Kepler's 2-3 Rule

$$T^2 = \frac{4\pi^2}{GM}r^3$$

$$(2.754\times10^4)^2 = \frac{4\pi^2}{(6.67\times10^{-11})M}(9.4\times10^6)^3$$

$$M = 6.48\times10^{23} \text{ kg}$$

13.47

$r = 2.2\times10^{20}$ m

$T = 2.5\times10^8$ y $= 7.85\times10^{15}$ s

$$T^2 = \frac{4\pi^2}{GM}r^3$$

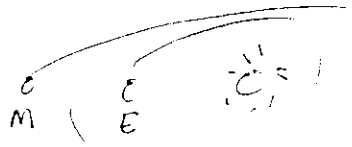
$$(7.85\times10^{15})^2 = \frac{4\pi^2}{(6.67\times10^{-11})M}(2.2\times10^{20})^3$$

$$M = 1.02\times10^{41} \text{ kg}$$

If each star is $\sim 2\times10^{30}$ kg, then there are

$$\frac{1.02\times10^{41}}{2\times10^{30}} \sim 5.1\times10^{10} \text{ stars}$$

13.48



$$T^2 = \frac{4\pi^2}{GM_{sun}} r^3$$

Form a ratio

$$\frac{T_m^2 = \frac{4\pi^2}{GM_{sun}} r_m^3}{T_E^2 = \frac{4\pi}{GM_{sun}} r_E^3}$$

$$\left(\frac{T_m}{T_E}\right)^2 = \left(\frac{r_m}{r_E}\right)^3 = (1.52)^3$$

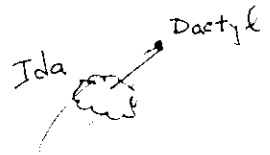
$$\frac{T_m}{T_E} = 1.87$$

$$T_m = 1.87\ T_E$$

$$T_m = 1.87 \text{ years}$$

Value in back of book is the same ~1.88 years

13.56



take $r \sim 100$ km

$T = 27h = 9.72 \times 10^4$ s

a) Use $T^2 = \frac{4\pi^2}{GM} r^3$

$$(9.72 \times 10^4)^2 = \frac{4\pi^2}{(6.67 \times 10^{-11})\, M} (100 \times 10^3)^3$$

$$M = 6.26 \times 10^{16} \text{ kg}$$

b) Volume $= 14100\ (\text{km})^3 = 1.41 \times 10^{13}\ \text{m}^3$

$$\text{density} = \frac{\text{mass}}{\text{vol}} = \frac{6.26 \times 10^{16}}{1.41 \times 10^{13}} = 4.44 \times 10^3\ \text{kg/m}^3$$

$$= 4.44\ \text{g/cm}^3$$