15.5

back & forth distance travel 2 mm

$f = 120\ \text{Hz}$ → $\omega = 2\pi f = 2\pi\, 120 = 753.6\ \frac{\text{rad}}{\text{sec}}$

$y = A \sin \omega t$

a) back & forth 2mm → $A = 1\ \text{mm}$

b) $V_y = \frac{dy}{dt} = \underbrace{\omega A}_{V_{max}} \cos \omega t$

$V_{max} = \omega A = 753.6\,(0.001) = 0.75\ \text{m/s}$

c) $a = \frac{dv}{dt} = \underbrace{-\omega^2 A}_{a_{max}} \sin \omega t$

$a_{max} = \omega^2 A = (753.6)^2 (0.001) = 568\ \text{m/s}^2$

15.9 $x = 6 \cos[3\pi t + \pi/3]$

$V = \frac{dx}{dt} = -3\pi 6 \sin[3\pi t + \pi/3]$

$a = \frac{dv}{dt} = -(3\pi)^2 6 \cos[3\pi t + \pi/3]$

a) At $t=2$ $x = 6 \cos[3\pi \cdot 2 + \pi/3] = 6 \cos[6\pi + \pi/3] = 6 \cos[\pi/3] = 3\ \text{m}$

b) At $t=2$ $V = -3\pi 6 \sin[3\pi \cdot 2 + \pi/3] = -3\pi 6 \sin[\pi/3] = -3\pi 6 \frac{\sqrt{3}}{2} = -48.9\ \text{m/s}$

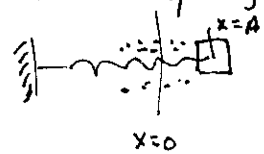
c) At $t=2$ $a = -(3\pi)^2 6 \cos[3\pi \cdot 2 + \pi/3] = -(3\pi)^2 6 \cos[\pi/3] = -(3\pi)^2 6\, \frac{1}{2} = -26.6\ \text{m/s}^2$

d) phase $\phi = \pi/3$

e) $\omega = 3\pi$ so that $\omega = 2\pi f$
$3\pi = 2\pi f$
$f = 1.5\ \text{Hz}$

f) Period $T = \frac{1}{f} = \frac{1}{1.5} = 0.67\ \text{sec}$

15.27 Visualize mass on spring



$E_{TOT} = KE + PE = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

when spring at maximum extent, all the energy is PE b/c $v=0$ $x=A$. So therefore $E_{TOT} = \frac{1}{2}m0^2 + \frac{1}{2}kA^2 = \frac{1}{2}kA^2$

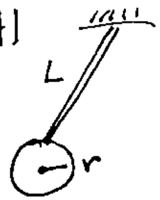
Energy is conserved in this system so that $E_{TOT} = \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ for any other $v + x$.

a) Suppose $x = \frac{1}{2}A$ $E_{TOT} = \frac{1}{2}kA^2 = KE + \frac{1}{2}k(\frac{A}{2})^2$
or $KE = \frac{1}{2}kA^2 \cdot \frac{3}{4}$
then $\frac{KE}{E_{TOT}} = \frac{\frac{1}{2}kA^2\ 3/4}{\frac{1}{2}kA^2} = 3/4$ or 75%

b) $\frac{PE}{E_{TOT}} = 1/4$ or 25%

c) At what displacement is the $PE = \frac{1}{2} E_{TOT}$?
$\frac{1}{2}kx^2 = \frac{1}{2}\left[\frac{1}{2}kA^2\right]$
→ $x = \frac{1}{\sqrt{2}}A$

15.41



disk $m = 0.5\,kg$ $r = 0.10\,m$
rod $M = 0.27\,kg$ $L = 5\,m$

$h = L + r = 5.1\,m$

a) $I_{Total} = I_{rod} + I_{shifted\ disk}$

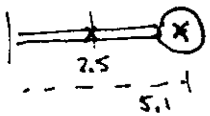
$$= \tfrac{1}{3}ML^2 + \left[\tfrac{1}{2}mr^2 + mh^2\right]$$

$$= \tfrac{1}{3}\,0.27(5)^2 + \left[\tfrac{1}{2}\,0.5(0.1)^2 + 0.5(5.1)^2\right]$$

$$= 2.248 + 2.5\times10^{-3} + 13.1$$

$$= 15.26\ kg\,m^2$$

b) Distance from pivot pt to cm



$$m_{Tot}\,x_{cm} = m_1x_1 + m_2x_2$$

$$0.77\,x_{cm} = 0.27(2.5) + 0.5(5.1)$$

$$x_{cm} = 4.19\ m$$

c) Period of oscillation

$T = \frac{2\pi}{\omega}$ with $\omega = \sqrt{\frac{mgh}{I}}$

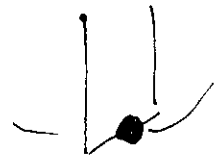
$$\omega = \sqrt{\frac{(0.77)(9.8)(4.19)}{15.26}}$$

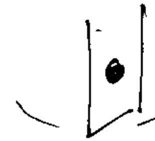
$$\omega = 1.44\ rad/s$$

$$T = \frac{2\pi}{1.44}$$

$$= 4.36\ sec$$

15.45





$T = 8.85\ sec$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$8.85 = 2\pi\sqrt{\frac{L}{9.8}}$$

$$L = 19.46\ m$$

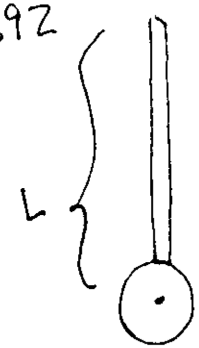
then $L \to 19.46 - 0.35$
$= 19.11\ m$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$= 2\pi\sqrt{\frac{19.11}{9.8}}$$

$$= 8.77\ sec$$

15.92



rod ℓ = ? ← Note curly ℓ
M = ~0 kg

disk
r = 0.15 m
m = 1.000 kg

$2\pi = 6.283185$

Q: If we were doing a high precision calculation, why would we ignore the rod mass?
~ I dunno.

Want $T = 2$ sec $g = 9.8\ m/s^2$

If you though to treat this as a simple pendulum, then just do

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$2 = 2\pi\sqrt{\frac{L}{9.8}}$$

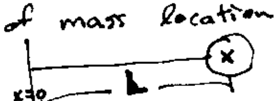
$$L = 0.992947\ m$$

Since we are asked to do this with high precision, we should probably calculate it as a "physical pendulum" too

$$I = \text{shifted disk} = \tfrac{1}{2}mr^2 + mh^2$$

$$I = \tfrac{1}{2}(1)(0.15)^2 + (1)L^2 = 0.01125 + L^2$$

The center of mass location will be



$$M_{Tot}\,x_{cm} = \cancel{M_1 x_1}^{\,0} + m_2 x_2$$

$$(1)\,x_{cm} = 1\,(L)$$

$$x_{cm} = L \quad \text{--- duh, of course}$$

Then

$$T = \frac{2\pi}{\omega} \qquad \omega = \sqrt{\frac{mgh}{I}}$$

15.96

"like a block on a spring"

$$\omega = \sqrt{\frac{k}{m}}$$

If mass m

$$\omega_m = \sqrt{\frac{k}{m}}$$

If mass 2.5m

$$\omega_{2.5m} = \sqrt{\frac{k}{2.5m}}$$

Then

$$\frac{f_{2.5}}{f_1} = \frac{\omega_{2.5}}{\omega_1} = \sqrt{\frac{1}{2.5}} = 0.632$$

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

$$2 = 2\pi\sqrt{\frac{0.01125 + L^2}{1\ 9.8\ L}}$$

$$\frac{1}{\pi^2} = \frac{0.01125 + L^2}{9.8L}$$

$$0.9929\,L = 0.01125 + L^2$$

$$0 = L^2 - 0.9929L + 0.01125$$

$$L = \frac{+0.9929 \pm \sqrt{(0.9929)^2 - 4(1)(0.01125)}}{2}$$

$$L = \frac{0.9929 \pm 0.96997}{2}$$

$L = 0.9814$ is the sensible solution
- slightly diff from before.

so that

$$\ell = L - 0.15 = 0.9814 - 0.15 = 0.8314\ m$$