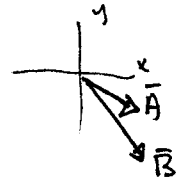
Ch 7.9 Angle btw vectors $\vec{A}\cdot\vec{B} = AB\cos\theta$

a) $\vec{A} = 3\hat{i} - 2\hat{j}$ $\quad$ $\vec{B} = 4\hat{i} - 4\hat{j}$

$A = \sqrt{13}$ $\quad$ $B = 4\sqrt{2}$



$\bar{A}\cdot\bar{B} = AB\cos\theta$

$12+8 = \sqrt{13}\,4\sqrt{2}\cos\theta$

$20 =$

$\theta = 11.3^\circ$

b) $\vec{A} = -2\hat{i} + 4\hat{j}$ $\quad$ $\vec{B} = 3i - 4j + 2\hat{k}$

$A = \sqrt{20}$ $\quad$ $B = \sqrt{29}$

$\bar{A}\cdot\bar{B} = AB\cos\theta$

$-6-16 = \sqrt{20}\sqrt{29}\cos\theta$

$\theta = 156^\circ$

c) $\vec{A} = \hat{i} - 2\hat{j} + 2\hat{k}$ $\quad$ $\vec{B} = 3j + 4\hat{k}$

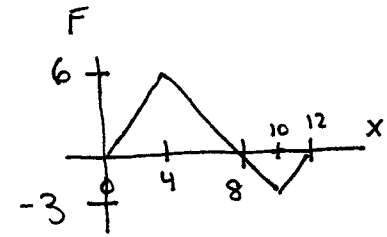
$A = \sqrt{9}$ $\quad$ $B = \sqrt{25}$

$\bar{A}\cdot\bar{B} = AB\cos\theta$

$-6+8 = \sqrt{9}\sqrt{25}\cos\theta$

$\theta = 82.3^\circ$

Ch 7.13



This problem asks for the area under the F(x) "curve".

a) Work by F btw 0–8 m

$\int_i^f \bar{F}\cdot d\bar{r} \longrightarrow \int_i^f F\,dx = \frac{1}{2}6\cdot 4 + \frac{1}{2}6\cdot 4$

$= 24$ Joules

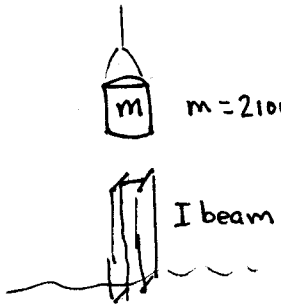
b) Work by F btw 8–10 m

$\int_i^f \bar{F}\cdot d\bar{r} \longrightarrow \int_i^f F\,dx = -\frac{1}{2}2\cdot 2 = -2$ Joules

c) Total work by F btw 0—10 m

$24 + -2 = 22$ Joules

Ch 7.33



The driver falls 5m before striking Ibeam

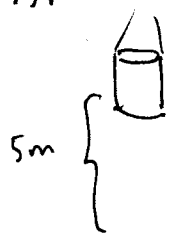
The Ibeam is driven 0.12m into the ground

There are two parts to the problem

A. As the mass falls

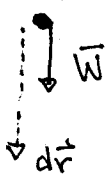
B. As the Ibeam is driven into ground

A.



The first part of the problem will tell us how fast the ▢ is moving just as it hits the Ibeam.

The only force on the ▢ is gravity



$\Delta r = 5$ m

Work by gravity = Change in KE

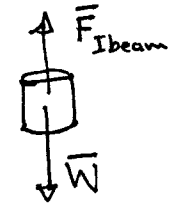
$\int \bar{W} \cdot d\bar{r}$

$mg \, \Delta r \cos 0^\circ = \frac{1}{2} m v_f^2 - \frac{1}{2} m \cancel{v_i^2}$ 0

$(2100)(9.8) \, 5 \cos 0^\circ = \frac{1}{2}(2100) v_f^2$

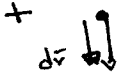
$v_f = 9.90$ m/s

B.



Work by F_{Ibeam} + Work by gravity = Change in KE

Work by F_{Ibeam} + $\int \bar{W} \cdot d\bar{r}$



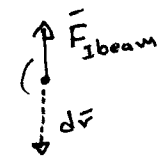
$\Delta r = 0.12$

Work by F_{Ibeam} + $mg \, \Delta r \cos 0^\circ = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

Work by F_{Ibeam} + $2100(9.8)0.12 = 0 - \frac{1}{2} 2100 (9.9)^2$

2469 −106910

Work by $F_{Ibeam} = -1.05 \times 10^5$



$\Delta r = 0.12$

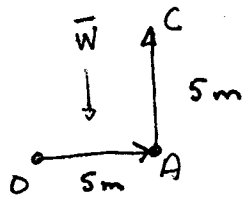
$\int \vec{F}_{Ibeam} \cdot d\vec{r} =$

$F_{Ibeam} \, \Delta r \cos 180^\circ =$

$F_{Ibeam} (0.12) \cos 180^\circ =$

$F_{Ibeam} = 8.78 \times 10^5$ Newtons

Ch 7.39

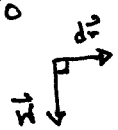


Work by $\vec{W} = \int_i^f \vec{W} \cdot d\vec{r}$

recall $|\vec{W}| = mg = 4(9.8)$
$= 39.2\,N$

a) Work by $\vec{W}$ going $O \to A \to C$

$$\int_O^A \vec{W} \cdot d\vec{r} + \int_A^C \vec{W} \cdot d\vec{r}$$



$\cdot$

$$\left(\begin{matrix}\text{zero b/c} \\ \theta = 90^\circ\end{matrix}\right) + \left(mg\,\Delta r \cos 180^\circ\right)$$

$$0 + 39.2\,(5)\,\cos 180^\circ$$

$$-196 \text{ Joules}$$

b) Work by $\vec{W}$ going $O \to B \to C$

$$\int_O^B \vec{W} \cdot d\vec{r} + \int_B^C \vec{W} \cdot d\vec{r}$$

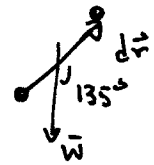
$$\left(mg\,\Delta r \cos 180^\circ\right) + \left(\begin{matrix}\text{zero b/c} \\ \theta = 90^\circ\end{matrix}\right)$$

$$39.2(5)\cos 180^\circ + 0$$

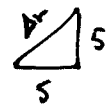
$$-196 \text{ Joules}$$

c) Work by $\vec{W}$ going $O \to C$

$$\int_O^C \vec{W} \cdot d\vec{r}$$



$$mg\,\Delta r \cos 135^\circ$$



$$\Delta r^2 = 5^2 + 5^2$$
$$= 50$$
$$\Delta r = \sqrt{50}$$

$$(39.2)\sqrt{50}\,\cos 135^\circ$$

$$-196 \text{ Joules}$$

Ch7.44 $\vec{F} = (-Ax + Bx^2)\ \hat{i}$ Newtons

a) Potential Energy Difference / Function

$$\Delta U = U(x) - U(0) = -\int_0^x F\,dx$$

$$= -\int_0^x (-Ax + Bx^2)\,dx$$

$$= \left(-\tfrac{1}{2}Ax^2 + \tfrac{1}{3}Bx^3\right]_0^x$$

$$U(x) - 0 = -\tfrac{1}{2}Ax^2 + \tfrac{1}{3}Bx^3$$

so

$$U(x) = -\tfrac{1}{2}Ax^2 + \tfrac{1}{3}Bx^3$$

b) Difference in potential energy $x=2 \longrightarrow x=3$

$$U(x=2) = -\tfrac{1}{2}A\,2^2 + \tfrac{1}{3}B\,2^3$$

$$= -2A + \tfrac{8}{3}B$$

$$U(x=3) = -\tfrac{1}{2}A\,3^2 + \tfrac{1}{3}B\,3^3$$

$$= -\tfrac{9}{2}A + 9B$$

$$\tfrac{27}{3} - \tfrac{8}{3} = \tfrac{19}{3}$$

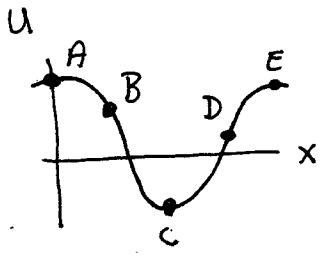
so

$$\Delta U = U(3) - U(2) = \left(-\tfrac{9}{2}A + 9B\right) - \left(-2A + \tfrac{8}{3}B\right)$$

$$= -\tfrac{5}{2}A + \tfrac{19}{3}B$$

Then $\Delta KE = +\tfrac{5}{2}A - \tfrac{19}{3}B$

Ch 7.47



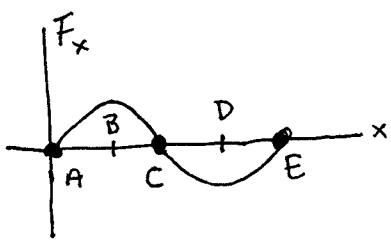
$$F_x = -\frac{dU}{dx}$$

a)
- (A) $F_x = 0$
- (B) F_x = positive
- (C) $F_x = 0$
- (D) F_x = negative
- (E) $F_x = 0$

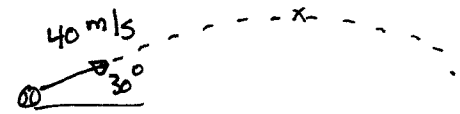
b) (A) & (E) unstable equilibrium

(C) stable equilibrium

c)



Ch 7.51



$V_{0x} = V_0 \cos 30° = 34.6$ m/s

$V_{0y} = V_0 \sin 30° = 20$ m/s

When the ball reaches its max height

$V_x = 20$ m/s and $V_y = 0$

so $|\vec{V}| = 20$ m/s

The kinetic energy at the max hgt is

$$\frac{1}{2}mv^2 = \frac{1}{2}(0.150)(20)^2$$

$$= 30 \text{ Joules}$$

Ch7.56 $\quad U = -x^3 + 2x^2 + 3x$

a) $F_x = -\frac{dU}{dx} = -\left[-3x^2 + 4x + 3\right]$

$= 3x^2 - 4x - 3$

b) when is $F_x = 0$?

$0 = 3x^2 - 4x - 3$

$x = \frac{+4 \pm \sqrt{16 - 4(3)(-3)}}{6}$

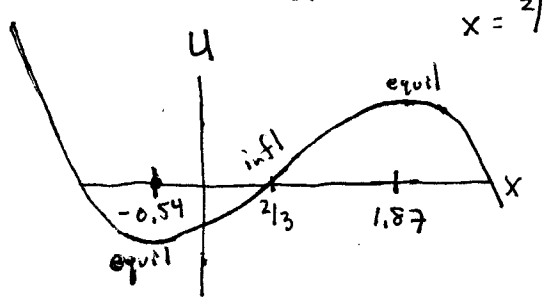
$= \frac{4 \pm \sqrt{16 + 36}}{6}$

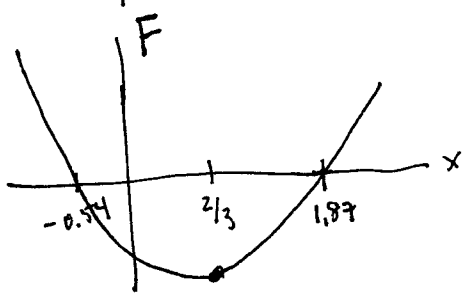
$= \frac{4 \pm 7.21}{6} = -0.54,\ 1.87$

c) when is $\frac{d^2U}{dx^2} = 0$?

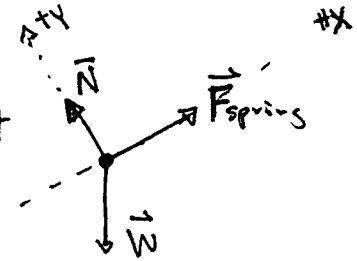
$\frac{d^2U}{dx^2} = 0 = 6x - 4$

$x = 2/3$



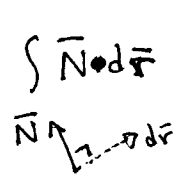


Ch7.57

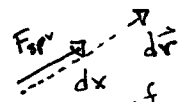


$k = 1.2\ \frac{N}{cm} = 120\ \frac{N}{m}$

Work by $\vec{N}$ $= \int \vec{N} \cdot d\vec{r} = N\,\Delta r \cos 90^\circ = 0$



Work by $\vec{F}_{spring}$ $= \int \vec{F}_{spring} \cdot d\vec{r}$



$= \int_i^f kx\ dx \cos 0^\circ$

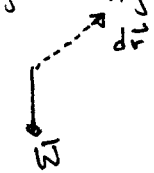
$= \frac{1}{2}kx^2 \Big]_i^f$

$= \frac{1}{2}k x_f^2 - \frac{1}{2}k x_i^2$

$= \frac{1}{2}\ 120\,(0.05)^2 - \frac{1}{2}\ 120\,(0.0)^2$

$= 0.15$ Joules

Work by Gravity $= \int \vec{W} \cdot d\vec{r}$ = it's only the force along the path that's important



$= -\ mg\sin\theta\ \Delta r$

$= -(0.1)\,9.8 \sin 10^\circ\,(0.05)$

$= -8.51 \times 10^{-3}$ Joules

Work by ALL forces = Change in KE

$0 + 0.15 + (-8.51 \times 10^{-3}) = \frac{1}{2}mv_f^2 - \cancel{\frac{1}{2}mv_i^2} \to 0$

$v_f = 1.68\ m/s$