9.6 

$\text{Impulse} = \Delta p = \int_i^f F\,dt$

$\text{Impulse} = \Delta p = F_{AVE}\,\Delta t$

$m\,\Delta v = F_{AVE}\,\Delta t$

$60\text{mph} = 26.8\ \text{m/s}$

$m = 12\ \text{kg}$

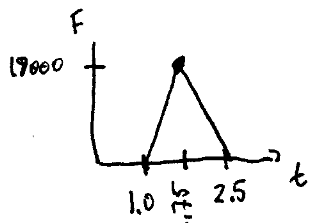
$12(26.8) = F_{AVE}(0.050\text{sec})$

$F_{AVE} = 6432$ Newtons

$= 1445$ lbs !

That's quite a big force that must be applied to retain the child in his arms.

9.7 Baseball & bat



c) Peak Force is 18000 Newtons

a) $\text{Impulse} = \Delta p = \int_{1\text{sec}}^{2.5\text{sec}} F\,dt$ = area under Λ

$= 2$ triangles

$= \frac{1}{2}(18000)\,0.75 + \frac{1}{2}(18000)(0.75)$

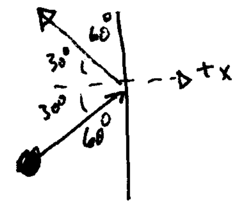
$= 1.35 \times 10^4\ \text{kg m/s}$

b) $\text{Impulse} = \Delta p = F_{AVE}\,\Delta t$

$1.35\times 10^4 = F_{AVE}\ 1.5$

$F_{AVE} = 9000\ \text{N}$

duh, could have written this down from the figure

9.9 

Only the x-component of the motion changes

$M = 2\ \text{kg}$

$\Delta t = 0.2\ \text{sec}$

$V_0 = 10\ \text{m/s}$

$\text{Impulse}_x = \Delta p_x = F_{AVE_x}\,\Delta t$

$p_{f_x} - p_{i_x} = F_{AVE_x}\,\Delta t$

$(\leftarrow) - (\rightarrow) =$

$(-m v \cos 30^\circ) - (m v \cos 30^\circ) =$

x-comp negative

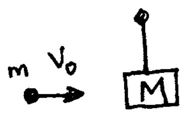
$-2 m v \cos 30^\circ = F_{AVE_x}\,\Delta t$

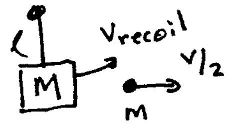
$-2(3)(10)\cos 30^\circ = F_{AVE_x}(0.2\text{sec})$

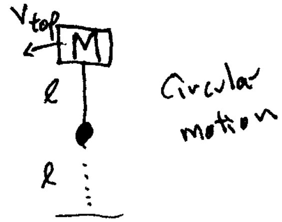
$F_{AVE_x} = -260$ Newtons

with $F_{AVE_y} = 0$ Newtons

9.18 Three pictures







Do the collision first (always)

$$\text{Init } P = \text{Final } P$$

$$m v_0 = M v_{recoil} + m\left(\frac{v_0}{2}\right)$$

so

$$\frac{1}{2} m v_0 = M v_{recoil}$$

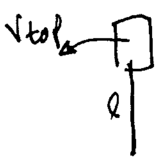
Now do the recoiling bob swinging up

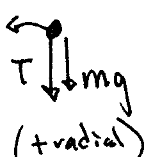
$$\text{Init Energy} = \text{Final Energy}$$

$$\frac{1}{2} M v_{recoil}^2 = \frac{1}{2} M v_{top}^2 + Mgh$$

$$= \frac{1}{2} M v_{top}^2 + Mg\,2\ell$$

Of course the circular motion constraints also have to be addressed





$$M a_{radial} = T + Mg$$

$$M \frac{v_{top}^2}{\ell} = T + Mg$$

the minimum speed would be when $T \to 0$

$$M \frac{v_{top}^2}{\ell} = 0 + Mg \qquad v_{top}^2 = g\ell$$

Now collecting what we have

$$\frac{1}{2} m v_0 = M v_{recoil}$$

$$\frac{1}{2} M v_{recoil}^2 = \frac{1}{2} M v_{top}^2 + Mg\,2\ell$$

$$v_{top}^2 = g\ell$$

Putting it together...

$$\frac{1}{2} m v_0 = M v_{recoil} \rightarrow \frac{1}{2}\frac{m}{M} v_0 = v_{recoil}$$

$$\frac{1}{2} M \left(\frac{1}{2}\frac{m}{M} v_0\right)^2 = \frac{1}{2} M g\ell + Mg\,2\ell$$

Multiply by 2, cancel big-M..

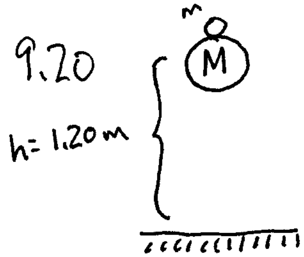
$$\left(\frac{1}{2}\frac{m}{M} v_0\right)^2 = g\ell + 4g\ell$$

$$= 5g\ell$$

$$\frac{1}{2}\frac{m}{M} v_0 = \sqrt{5g\ell}$$

$$v_0 = 2\frac{M}{m}\sqrt{5g\ell}$$

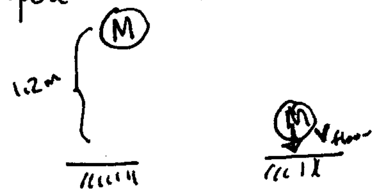
9.20



$m = 0.057$ kg
$M = 0.590$ kg

$M_{TOT} = 0.647$ kg

a) Speed of basketball at floor



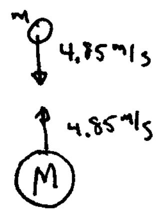
Init Energy = Final Energy

$$Mgh = \frac{1}{2} M v_{floor}^2$$

$$9.8(1.2) = \frac{1}{2} v_{floor}^2$$

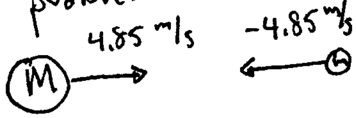
$$v_{floor} = 4.85 \text{ m/s}$$

b) Collision



the tennis ball will have same speed because it falls the same distance.

reDraw this sideways for convienience in working problem



Follow our procedure

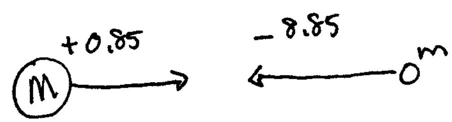
$$M_{TOT} v_{cm} = M_1 v_1 + M_2 v_2$$

$$0.647\, v_{cm} = 0.590(4.85) + 0.057(-4.85)$$

$$v_{cm} = 4.00 \text{ m/s}$$

subtract v_{cm}

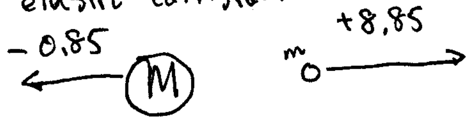
cm reference frame



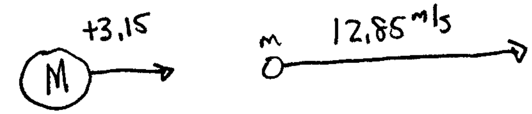
[continue next column]

Do the elastic collision

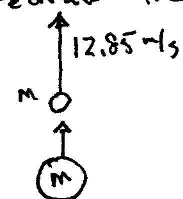
cm reference frame



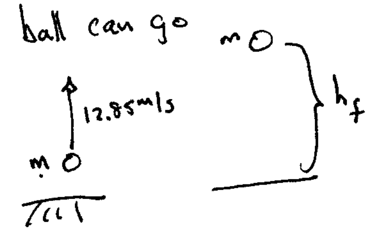
Add v_{cm}



Now redraw the problem up & down.



- And calculate how high the tennis ball can go



Init Energy = Final Energy

$$\frac{1}{2} \cancel{m} v^2 = \cancel{m} g h$$

$$\frac{1}{2} (12.85)^2 = 9.8\, h$$

$$h = 8.4 \text{ meters}$$

~ 26 ft

9.23 We are doing this with neutrons striking hydrogen. The neutron and hydrogen atom have very similar masses, so I'll pretend they are the same.

m v_0 → m

Follow our procedure

$$M_{Tot} v_{cm} = m_1 v_1 + m_2 v_2$$
$$(2m) v_{cm} = m v_0 + m(0)$$
$$v_{cm} = \tfrac{1}{2} v_0$$

Subtract v_{cm}

m $\tfrac{1}{2}v_0$ → ← $-\tfrac{1}{2}v_0$ m } cm reference frame

Do elastic collision

← $-\tfrac{1}{2}v_0$ m m $+\tfrac{1}{2}v_0$ → } cm reference frame

Add v_{cm}

m m v_0 →

Notice that the incoming neutron stops!

a) All energy is transfered to the hydrogen atom

b) All of it

This is why water is used as a moderator and heat transfer fluid in reactors.

We could work this as specified in the book with neutrons → Carbon

m v_0 → $12m$

$$M_{Tot} v_{cm} = m_1 v_1 + m_2 v_2$$
$$(13m) v_{cm} = m v_0 + (12m)(0)$$
$$v_{cm} = \tfrac{1}{13} v_0$$

Subtract v_{cm}

m $\tfrac{12}{13}v_0$ → ← $-\tfrac{1}{13}v_0$ $12m$ } cm reference frame

Do elastic collision

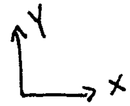
← $-\tfrac{12}{13}v_0$ $+\tfrac{1}{13}v_0$ → } cm reference frame

Add v_{cm}

← $-\tfrac{11}{13}v_0$ m $12m$ $\tfrac{2}{13}v_0$ →

Then can answer the questions by looking at the pictures above

9.26



a) A perfectly inelastic collision wastes the most energy, however in view of our previous problem with a neutron, a specially designed elastic collision may not be so bad.

b) After collision, only cm motion remains.

$$\overrightarrow{\text{Init Momentum}} = \overrightarrow{\text{Final Momentum}}.$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1+m_2)\vec{V}_{cm}$$

$$90(+5\hat{i}) + 95(+3\hat{j}) = 185\,\vec{V}_{cm}$$

$$\vec{V}_{cm} = 2.43\,\hat{i} + 1.54\,\hat{j} \quad m/s$$

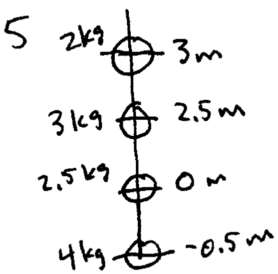
$$|\vec{V}_{cm}| = 2.88 \ m/s$$

c) Energy Loss

before	after
$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$	$\frac{1}{2}(m_1+m_2)v_{cm}^2$
$\frac{1}{2}90(5)^2 + \frac{1}{2}95(3)^2$	$\frac{1}{2}185(2.88)^2$
1553 Joules	766 Joules

Energy Lost = 787 Joules into things that happen between the players.

9.35

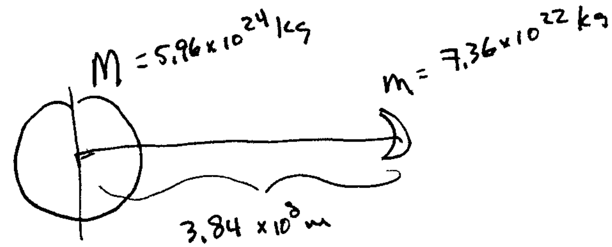


$$M_{TOT}\,y_{cm} = m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4$$

$$11.5\,y_{cm} = 2(3) + 3(2.5) + 2.5(0) + 4(-0.5)$$

$$y_{cm} = 1.0 \ m$$

9.36



$$M_{TOT}\,x_{cm} = m_1x_1 + m_2x_2$$

$$(7.36\times10^{22} + 5.96\times10^{24})\,x_{cm} = 5.96\times10^{24}(0) + 7.36\times10^{22}(3.84\times10^{8})$$

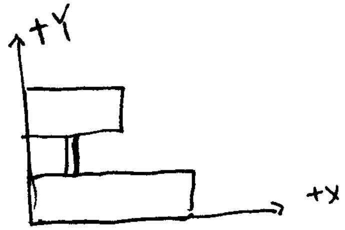
$$x_{cm} = 4.68\times10^{6} \ m$$

$$= 4680 \ km$$ above center of Earth.

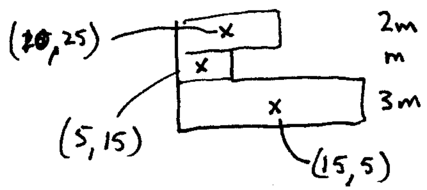
The Earth-Moon system actually revolves about this point

The Earth's radius is 6370 km

9.37



There are a couple of ways to approach this, the easiest may be to pretend there are 3 rows of material & then locate the cm of each row — then put it together



$M_{TOT}\, x_{cm} = m_1 x_1 + m_2 x_2 + m_3 x_3$

$6m\ x_{cm} = (3m)\,15 + m\,(5) + 2m(10)$

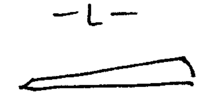
$x_{cm} = 11.7$ cm

$M_{TOT}\, y_{cm} = m_1 y_1 + m_2 y_2 + m_3 y_3$

$6m\ y_{cm} = (3m)(5) + m(15) + 2m\,(25)$

$y_{cm} = 13.3$ cm

9.39



$L = 30$ cm $= 0.3$ m

$\lambda(x) = 50 + 20x$

$$\text{Total Mass} = \int_{L\,end}^{R\,end} dm = \int_{L\,end}^{R\,end} \lambda\, dx$$

$$= \int_0^{0.3} (50 + 20x)\,dx$$

$$= \left(50x + \tfrac{1}{2}20x^2\right]_0^{0.3}$$

$$= 50(0.3) + \tfrac{1}{2}20\,(0.3)^2$$

$$= 15.9 \text{ gm}$$

Center of Mass

$$M_{TOT}\, x_{cm} = \int dm\, x$$

$$= \int \lambda\, dx\ x$$

$$= \int \lambda x\ dx$$

$$= \int (50 + 20x)\ x\ dx$$

$$= \int_0^{0.3} (50x + 20x^2)\,dx$$

$$= \tfrac{1}{2}50x^2 + \tfrac{1}{3}20x^3 \Big]_0^{0.3}$$

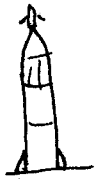
$$= \tfrac{1}{2}50(0.3)^2 + \tfrac{1}{3}20(0.3)^3$$

$M_{TOT}\ x_{cm} = 2.43$

$15.9\ x_{cm} = 2.43$

$x_{cm} = 0.153$ m

9.51



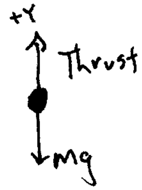
$\frac{dm}{dt} = 1.50 \times 10^4$ kg/s

$v_{exhaust} = 2.60 \times 10^3$ m/s

a) Thrust $= \frac{dm}{dt} v_e = (1.50\times10^4)/(2.60\times10^3)$

$= 3.9 \times 10^7$ Newtons

b)



$ma_y = \text{Thrust} - mg$

$(3\times10^6)a_y = [3.9\times10^7] - (3\times10^6)(9.8)$

$a_y = 3.2$ m/s^2

9.57 Two part problem: Collision & Projectile.

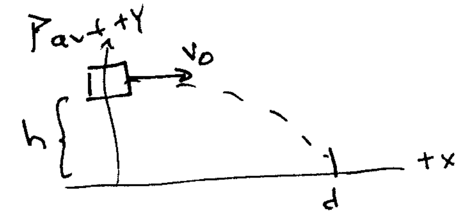
Collision Part

Init Momentum = Final Momentum

$mv_i = (m+M)\,v_o$

(Continue)

Projectile Part



$x = x_o + v_{ox}t + \frac{1}{2}a_x t^2$

$x_o = 0$

$v_{ox} = v_o$

$a_x = 0$

$x = v_o t$

$y = y_o + v_{oy}t + \frac{1}{2}a_y t^2$

$y_o = h$

$v_{oy} = 0$

$a_y = -9.8 = -g_{e\ 9.8}$

$y = h + \frac{1}{2}(-g)t^2$

block on ground when $y = 0$

$0 = h + \frac{1}{2}(-g)t^2$

$t = \sqrt{\frac{2h}{g}}$

distance from table

$d = v_o\sqrt{\frac{2h}{g}}$

$v_o = d\sqrt{\frac{g}{2h}}$

Then plug this into the earlier expression

$mv_i = [m+M]\left[d\sqrt{\frac{g}{2h}}\right]$

$v_i = \frac{m+M}{m}\, d\sqrt{\frac{g}{2h}}$

not terribly pretty, but it's OK.