10.1

$\theta = 5 + 10t + 2t^2$

$\theta(t) = 5 + 10t + 2t^2$

$\omega(t) = \frac{d\theta}{dt} = 10 + 4t$

$\alpha(t) = \frac{d\omega}{dt} = 4$

at $t = 0$

$\theta = 5$ rad

$\omega = 10$ rad/sec

$\alpha = 4$ rad/sec^2

at $t = 3$ s

$\theta = 5 + 10(3) + 2(3)^2 = 53$ rad

$\omega = 10 + 4(3) = 22$ rad/s

$\alpha = 4$ rad/s^2

10.3

starts at rest
reaches 12 rad/s in 3 sec

a) $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{12}{3} = 4$ rad/s^2

b) $\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\theta_0 = 0$

$\omega_0 = 0$

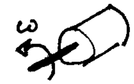
$\alpha = 4$

$\theta(t) = \frac{1}{2} 4 t^2$

at $t = 3$ sec

$\theta = \frac{1}{2} 4(3)^2 = 18$ rad

10.5



$\omega_0 = 100 \frac{\text{rev}}{\text{min}} = 100 \frac{2\pi \text{ rad}}{60 s} = 10.47$ rad/s

$\alpha = -2$ rad/s^2

$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$= 10.47t + \frac{1}{2}(-2)t^2$

$\omega(t) = \omega_0 + \alpha t$

$= 10.47 + (-2)t$

a) Comes to rest

$\omega = 10.47 - 2t$

$0 = 10.47 - 2t$

$t = 5.24$ s

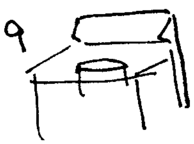
b) $\theta = 10.47t + \frac{1}{2}(-2)t^2$

$= 10.47(5.24) + \frac{1}{2}(-2)(5.24)^2$

$= 27.41$ rad

$= 4.36$ rev

10.9



Spinning Up
(8 sec)

$\omega_0 = 0$

$\omega_f = 5$ rev/s $= 5\ 2\pi$/s $= 31.4$ rad/s

$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{31.4}{8} = 3.93$ rad/s^2

$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$= \frac{1}{2}(3.93)t^2$

$= \frac{1}{2}(3.93)(8)^2 = 126$ rad

Spinning Down
(12 sec)

$\omega_0 = 31.4$ rad/s

$\alpha = \frac{-31.4}{12} = -2.62$ rad/s^2

$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$= 31.4t + \frac{1}{2}(-2.62)t^2$

$= 31.4(12) + \frac{1}{2}(-2.62)\,12^2 = 188$ rad

Total rotation $= 126 + 188 = 314$ rad $= 5$ revolutions

10.11 Estimate # revolutions auto tire turns in a year

Suppose take tire to be P275/55R20

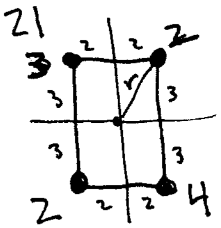
→ online calculator says the circumference is 100.25 inches

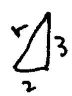
Distance traveled in 1 yr ~ 20k miles

~ 1.26×10^9 inches

revolutions = 1.26×10^7 revolutions

10.21





$r^2 = 2^2 + 3^2$

$r^2 = 4 + 9 = 13$

$r = \sqrt{13}$

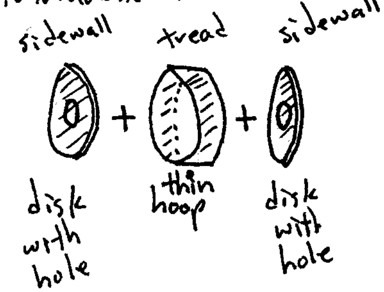
$I = \sum m_i r_i^2 = 3(13) + 2(13) + 4(13) + 2(13)$

$= 11(13) = 143 \text{ kgm}^2$

$\omega = 6 \text{ rad/s}$

rot $KE = \frac{1}{2} I \omega^2 = \frac{1}{2}(143)6^2 = 2.57 \times 10^3$ Joules

10.27 Automobile tire



Assume we aren't supposed to include the rim, only the rubber tire

The contribution from the thin hoop is

$\sim M R^2$, $R \sim 0.33$ m

$M \sim \rho \cdot 2\pi R \cdot \text{thickness} \cdot \text{width}$

$\sim (1.1 \times 10^3)[2\pi\, 0.33](0.0250)(0.20)$

~ 11.4 kg

$\sim 11.4\,(0.33)^2$

$\sim 1.24 \text{ kg m}^2$

One side wall can be calculated as a disk minus a central hole

$\frac{1}{2} m R^2$

$m \sim \rho\, \pi R^2$ thickness

$\sim (1.1 \times 10^3)\, \pi\, (0.305)^2 (0.00635)$

~ 2.04

then $\frac{1}{2} M R^2 \sim \frac{1}{2}[2.04](0.305)^2$

9.49×10^{-2}

$\frac{1}{2} m R^2$, $R = 0.165$ m

$m \sim \rho\, \pi R^2$ thickness

$\sim (1.1 \times 10^3)\, \pi\, (0.165)^2 (0.00635)$

~ 0.597 kg

$\frac{1}{2}[0.597](0.165)^2$

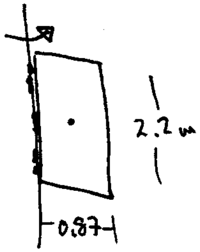
$8.13 \times 10^{-3} \text{ kg m}^2$

The whole moment of inertia is then

$I = I_{tread} + 2 I_{sidewall} = 1.24 + 2\left[9.49 \times 10^{-2} - 8.13 \times 10^{-3}\right]$

$= 1.41 \text{ kg m}^2$

10.28

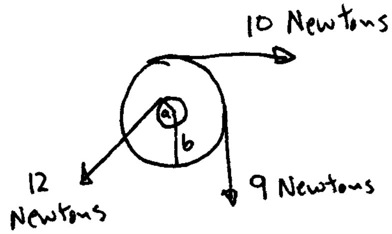


mass = 23 kg

Because the distance along the rotation axis doesnt appear in the formula (i.e. the height)
We can use the rod-about--an-end formula

$$I = \frac{1}{3} M L^2$$
$$= \frac{1}{3}\, 23\, (0.87)^2$$
$$= 5.80 \text{ kg m}^2$$

10.33



$a = 0.10$ m
$b = 0.25$ m
Call CW (+) Torque

Note that all forces are actually $\perp$ to their "lever arms"

$$\tau = 10\,b + 9\,b - 12\,a$$
$$= 10(0.25) + 9(0.25) - 12(0.10)$$
$$= 3.55 \text{ Newton-meters}$$

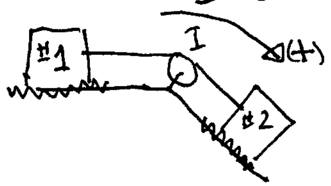
10.37

$m_1 = 2\ \text{kg}$

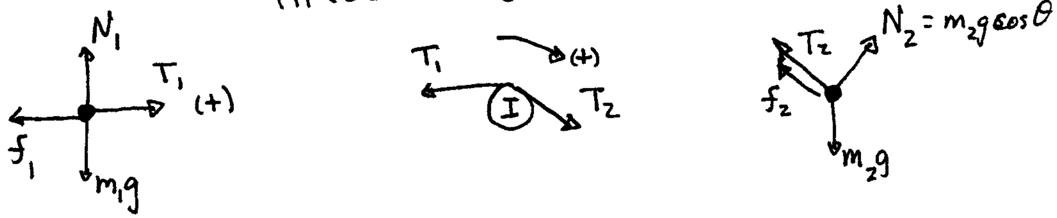
$m_2 = 6\ \text{kg}$

pulley $M = 10$ $R = 0.25$

$I = \frac{1}{2} M R^2 = 0.3125\ \text{kgm}^2$



THREE OBJECTS



$$m_1 a = T_1 - f_1$$
$$= T_1 - \mu N_1$$
$$= T_1 - \mu m_1 g$$

$$I\alpha = T_2 R - T_1 R$$

No slipping $a = R\alpha$

$$\frac{I}{R^2} a = T_2 - T_1$$

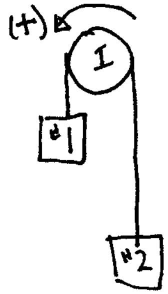
$$m_2 a = m_2 g \sin\theta - f_2 - T_2$$
$$= m_2 g \sin\theta - \mu N_2 - T_2$$
$$= m_2 g \sin\theta - \mu m_2 g \cos\theta - T_2$$

To get an expression for the acceleration quickly, add all 3 eqns

$$\left(m_1 + m_2 + \frac{I}{R^2}\right) a = -\mu m_1 g + m_2 g \sin\theta - \mu m_2 g \cos\theta$$

After we substitute values and solve for 'a', the tensions are obtained by plugging into the equations above.

10.44



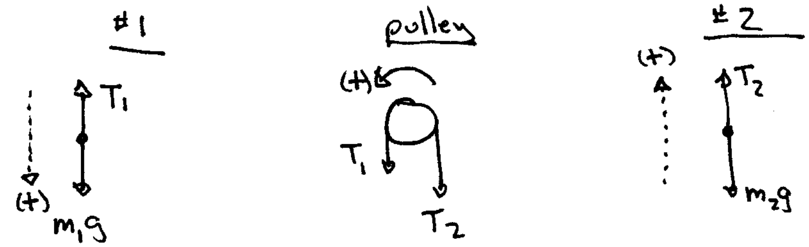
$m_1 = 20$ kg

$m_2 = 12.5$ kg

pulley $M = 5$ kg $R = 0.20$

$I = \frac{1}{2}MR^2 = 0.10$ kgm^2

THREE OBJECTS



$m_1 a = m_1 g - T_1$

$I\alpha = T_1 R - T_2 R$

$m_2 a = T_2 - m_2 g$

No slipping $a = R\alpha$

$\frac{I}{R^2} a = T_1 - T_2$

To get acceleration, add all equations

$$\left[m_1 + m_2 + \frac{I}{R^2}\right] a = m_1 g - m_2 g$$

$$a = \frac{(m_1 - m_2)\,g}{\left[m_1 + m_2 + I/R^2\right]}$$

$$a = \frac{(20 - 12.5)\,9.8}{20 + 12.5 + 0.10/0.2^2}$$

$$a = 2.1 \text{ m/s}^2$$

to travel 4 m, starting from rest, constant accel

$$y = \frac{1}{2} a t^2$$

$$4 = \frac{1}{2}(2.1)\,t^2$$

$$\rightarrow t = 1.95 \text{ sec}$$

If it was an ideal pulley, $I \rightarrow 0$

$$a = \frac{(20 - 12.5)\,9.8}{20 + 12.5 + 0} = 2.26$$

and then

$$y = \frac{1}{2} a t^2$$

$$4 = \frac{1}{2}\, 2.26\, t^2$$

$$\rightarrow t = 1.88 \text{ sec}$$

10.79

Assume $r << R$

Total Init Energy = Total Final Energy

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$h = R - R\cos\theta$

$= R(1-\cos\theta)$

solid ball $I = \frac{2}{5}mr^2$

$$mgR(1-\cos\theta) = \frac{1}{2}mv^2 + \frac{1}{2}\frac{2}{5}mr^2\omega^2$$

$v = r\omega$

$$mgR(1-\cos\theta) = \frac{1}{2}mr^2\omega^2 + \frac{1}{2}\frac{2}{5}mr^2\omega^2$$

$$mgR(1-\cos\theta) = \left[\frac{1}{2}mr^2 + \frac{1}{5}mr^2\right]\omega^2$$

$\frac{5}{10} + \frac{2}{10}$

$$\cancel{m}gR(1-\cos\theta) = \frac{7}{10}\cancel{m}r^2\omega^2$$

$$\omega^2 = \frac{10}{7}\frac{gR(1-\cos\theta)}{r^2}$$