11.1 $\vec{M} = 6\hat{i} + 2\hat{j} - \hat{k}$

$\vec{N} = 2\hat{i} - \hat{j} - 3\hat{k}$

$$\bar{M} \times \bar{N} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 2 & -1 \\ 2 & -1 & -3 \end{vmatrix}$$

$$= \hat{i}(-6-1) - \hat{j}(-18 - {}^{-}2) + \hat{k}(-6-4)$$

$$= -7\hat{i} + 16\hat{j} - 10\hat{k}$$

11.3 $\vec{A} = -3\hat{i} + 4\hat{j}$ $\quad \vec{B} = 2\hat{i} + 3\hat{j}$

a) $$\bar{A} \times \bar{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 4 & 0 \\ 2 & 3 & 0 \end{vmatrix} = \hat{i}\,0 - \hat{j}\,0 + \hat{k}(-9-8)$$

$$= 0\hat{i} + 0\hat{j} - 17\hat{k}$$

b) Angle $|\bar{A} \times \bar{B}| = A B \sin\phi$

$$17 = 5\sqrt{13}\sin\phi$$

$$\phi = 70.6^\circ$$

$A = \sqrt{(-3)^2 + 4^2}$
$= 5$

$B = \sqrt{2^2 + 3^2}$
$= \sqrt{13}$

11.24 a) Earth

$m = 5.96 \times 10^{24}$ kg

$R = 6370$ km $= 6.370 \times 10^6$ m

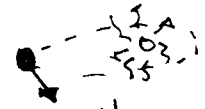
$$\omega = \frac{2\pi}{T} = \frac{2\pi}{24\text{hrs}} = \frac{2\pi}{86400\text{ s}} = 7.27 \times 10^{-5}\ \text{rad/s}$$

solid ball

$$I = \tfrac{2}{5}MR^2$$

$$L = I\omega = \tfrac{2}{5}MR^2 = \tfrac{2}{5}(5.96\times10^{24})[6.370\times10^6]^2(7.27\times10^{-5}\ \text{rad/s})$$

$$= 4.48 \times 10^{40}\ \text{kg m}^2/\text{s}$$

b) 

The Earth is effectively a point object. We could use the formula $L = rp$ or $L = I\omega$. I think I'll use $I\omega$ because it's perhaps slightly quicker.

$$I = mR^2 \qquad \omega = \frac{2\pi}{1\text{year}} = \frac{2\pi}{3.14\times10^7\text{ s}}$$

$$= 2\times10^{-7}\ \text{rad/s}$$

$m = 5.96\times10^{24}$ kg

$R = 1.5\times10^{11}$ m (fact of book)

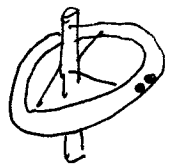
$$L = I\omega = mR^2\omega = (5.96\times10^{24})[1.5\times10^{11}]^2(2\times10^{-7})$$

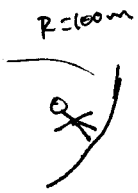
$$= 2.68\times10^{40}\ \text{kg m/s}$$

c) Interesting that they are roughly the same size.

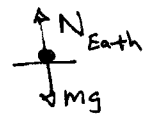
11.27

a)





The people walking on the ring should have the same sensation as walking on the Earth. This means that the normal force they experience should be the same as if they were standing on Earth



$N_{Earth} = mg$

$$m a_{radial} = \sum F_{rad}$$

$$m \frac{v^2}{R} = N_{space\ station}$$

we want $N_{space} = N_{Earth} = mg$

$$m \frac{v^2}{R} = mg$$

$$v^2 = gR \rightarrow v = \sqrt{(9.8)(100m)}$$

$$= 31.3\ m/s$$

The angular speed would be

$$v = R\omega$$

$$31.3 = 100\,\omega$$

$$\omega = 0.313\ rad/s$$

Angular Momentum

$$L = I\omega = MR^2\,\omega = (5\times10^4 kg)[100]^2\,0.313$$

thin cylinder

$$= 1.565\times10^8\ kg\,m^2/s$$

b) How long to fire rockets?

$$I\alpha = \sum \tau \qquad \vec{\tau} = \vec{r}\times\vec{F}$$

$$I\alpha = 2RF$$

$$MR^2\,\alpha = 2RF \qquad \text{Thrust} = 150\ \text{Newtons}$$

$$(5\times10^4)(100)^2\,\alpha = 2(100)(150)$$

$$\alpha = 6.0\times10^{-5}\ rad/s$$

then

$$\omega = \cancel{\omega_0}^{\,0} + \alpha t$$

$$0.313 = (6\times10^{-5})\,t$$

$$t = 5.22\times10^3\ sec$$

c) Now

$$\frac{dL}{dt} = \sum\tau$$

$$\frac{\Delta L}{\Delta t} = \sum\tau$$

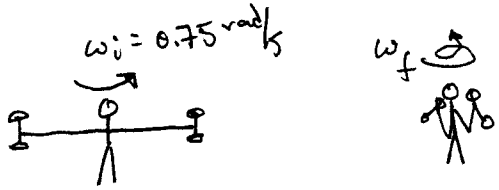
$$\Delta L = (\sum\tau)\,\Delta t$$

$$= 2RF\ \Delta t$$

$$1.5656\times10^8 \stackrel{?}{=} [2\cdot100\cdot150](5.22\times10^3)$$

$$1.5656\times10^8 \stackrel{?}{=} 1.565\times10^8 \checkmark$$

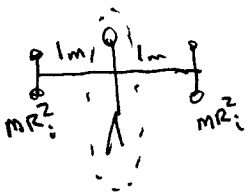
11.32



a)

Init Ang Mom = Final Ang Mom

$$I_{init}\,\omega_{init} = I_{final}\,\omega_{final}$$



$I_{init} = I_{student} + I_{dumb\,bell} + I_{dumb\,bell}$ $\qquad$ $I_{final} = I_{student} + I_{tube} + I_{dumbell}$

$$\left[I_{stud} + 2I_{dumb\,bell}\right]\omega_{init} = \left[I_{stud} + 2I_{dumb}\right]\omega_{final}$$

$$\left[I_{stud} + 2MR_i^2\right]\omega_{init} = \left[I_{stud} + 2MR_f^2\right]\omega_{final}$$

$$\left[\underbrace{3 + 2(3)(1)^2}_{9}\right](0.75\,\tfrac{rad}{s}) = \left[\underbrace{3 + 2(3)(0.3)^2}_{3.54}\right]\omega_{final}$$

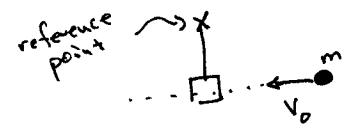
$$\omega_{final} = 1.91\ rad/sec$$

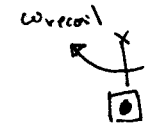
b) Kinetic Energies

$$KE_i = \tfrac{1}{2}I_i\omega_i^2 = \tfrac{1}{2}(9)(0.75)^2 = 2.53\ \text{Joules}$$

$$KE_f = \tfrac{1}{2}I_f\omega_f^2 = \tfrac{1}{2}(3.54)(1.91)^2 = 6.46\ \text{Joules}$$

11.37 Collision occurs so use conservation of momentum. In this case angular momentum, since rotations are involved.





treat as point object of mass $m+M$

Init L = Final L

calculate the initial L using the impact parameter $L = rP$

$$\ell m V_0 = I\,\omega_{recoil}$$

$$\ell m V_0 = (m+M)\,\ell^2\,\omega_{recoil}$$

$$\omega = \frac{m}{m+M}\frac{1}{\ell}V_0$$

Initial KE

$$KE_i = \tfrac{1}{2}mV_0^2$$

Final KE

$$KE_f = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left[(m+M)\ell^2\right]\left[\frac{m}{m+M}\frac{1}{\ell}V_0\right]^2$$

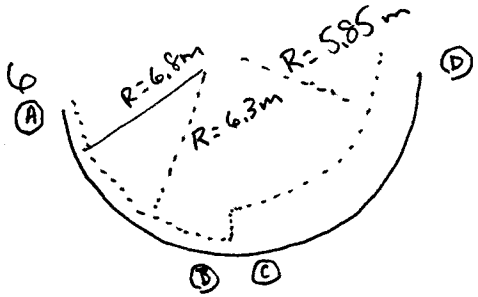
$$= \tfrac{1}{2}\frac{m^2}{m+M}V_0^2$$

$$= \tfrac{1}{2}\frac{m}{m+M}mV_0^2$$

Can write

$$KE_f = \frac{m}{m+M}KE_i$$

11.46



$m = 76\,kg$

a) Speed at B

$$\cancel{m}gh = \tfrac{1}{2}\cancel{m}v_B^2$$

$$9.8(6.3) = \tfrac{1}{2}v_B^2$$

$$v_B = 11.1\ m/s$$

b) Ang Momentum about center of curvature while at B

$$L = rp = rmv$$
$$= 6.3(76)(11.1)$$
$$= 5.31\times10^3\ kg\,m^2/s$$

c) Ang Momentum is constant because no Torque.

The forces are parallel & antiparallel to $\vec{r}$

d) Init L = Final L

$$r_i p_i = r_f p_f$$

$$r_i \cancel{m} v_i = r_f \cancel{m} v_f$$

$$6.3(11.1) = 5.85\,v_f$$

$$v_f = 11.95\ m/s$$

e) Change in kinetic energy

$$KE_i = \tfrac{1}{2}mv^2 = \tfrac{1}{2}76(11.1)^2 = 4.68\times10^3\ \text{Joules}$$

$$KE_f = \tfrac{1}{2}mv_f^2 = \tfrac{1}{2}76(11.95)^2 = 5.43\times10^3\ \text{Joules}$$

$$\Delta KE = 0.75\times10^3 = 750\ \text{Joules}$$

f) Speed at D

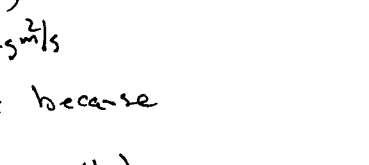
Tot Energy C = Total Energy D

$$\tfrac{1}{2}\cancel{m}v_C^2 = \tfrac{1}{2}\cancel{m}v_D^2 + \cancel{m}gh$$

$$\tfrac{1}{2}(11.95)^2 = \tfrac{1}{2}v_D^2 + g(5.85)$$

$$v_D = 5.30\ m/s$$

g) How high?



work this with conserv energy reseting zero pot'l energy at top of 1/2-pipe

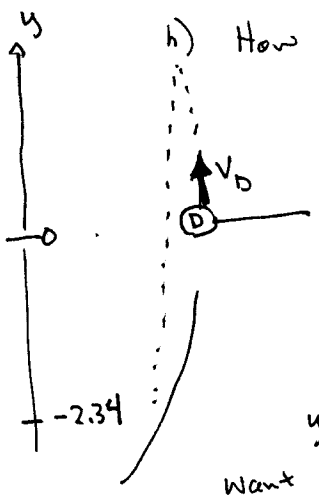
$$\tfrac{1}{2}\cancel{m}v_D^2 = \cancel{m}gh$$

$$\tfrac{1}{2}(5.3)^2 = 9.8h$$

$$h = 1.44\ m$$

h) next page.

11.46 cont



h) How long airborne?

$$y(t) = y_0 + v_{oy}t + \tfrac{1}{2}a_y t^2$$

$y_0 = 0$

$v_{oy} = 5.30 \text{ m/s}$

$a_y = -9.8 \text{ m/s}^2$

$$y(t) = 5.30t + \tfrac{1}{2}(-9.8)t^2$$

want to know

$$-2.34 = 5.30t + \tfrac{1}{2}(-9.8)t^2$$

$$4.9t^2 - 5.30t - 2.34 = 0$$

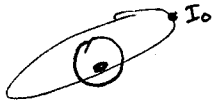
$$t = \frac{5.30 \pm \sqrt{(5.30)^2 - 4(4.9)(-2.34)}}{2(4.9)}$$

$$t = \frac{5.30 \pm 8.60}{9.8} = \cancel{-0.5},\ 1.42 \text{ sec}$$

$$t = 1.42 \text{ sec}$$

this problem just goes on forever.

13.15 Io, satellite of Jupiter



$$T^2 = \frac{4\pi^2}{GM} r^3$$

$T = 1.77\,d$
$= 1.53 \times 10^5\,s$

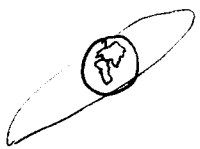
$r = 4.22 \times 10^5\,km$
$= 4.22 \times 10^8\,m$

$G = 6.67 \times 10^{-11}\,\frac{Nm^2}{kg^2}$

$$(1.53 \times 10^5)^2 = \frac{4\pi^2}{GM}(4.22 \times 10^8)^3$$

$$M = 1.90 \times 10^{27}\,kg$$

13.17 "Geostationary" Synchronous satellite above earth.



$$T^2 = \frac{4\pi^2}{GM} r^3$$

want
$T = 1\,day$
$= 86400\,sec$

$M = 5.96 \times 10^{24}\,kg$

$$(86400)^2 = \frac{4\pi^2}{GM} r^3$$

$r = 4.221 \times 10^7\,m$
$= 4.221 \times 10^4\,km$
$= 42200\,km$

or at an altitude of

$$\begin{array}{r} 42200 \\ -\ 6370 \\ \hline 35840\,km \end{array}$$

This orbit tends to be used for communications satellites

Geosynchronous orbit

From Wikipedia, the free encyclopedia

A **geosynchronous orbit** is an orbit around the Earth with an orbital period matching the Earth's sidereal rotation period[1]. This synchronization means that for an observer at a fixed location on Earth, a satellite in a geosynchronous orbit returns to exactly the same place in the sky at exactly the same time each day. In principle, any orbit with a period equal to the Earth's rotational period is technically geosynchronous; however, the term is often used[2] to refer to the special case of a geosynchronous orbit that is circular (or nearly circular) and at zero (or nearly zero) inclination, that is, directly above the equator. This is customarily called a geostationary orbit.

A **semisynchronous orbit** has an orbital period of 0.5 sidereal days, i.e. 11 h 58 min. Relative to the Earth's surface it has twice this period, and hence appears to go around the Earth once every day. Examples include the Molniya orbit and the orbits of the satellites in the Global Positioning System.

Contents

- 1 Orbital characteristics
- 2 Geostationary orbit
- 3 Synchronous orbits around general astronomical objects
- 4 Other geosynchronous orbits
- 5 History
- 6 See also
- 7 References
- 8 External links

Orbital characteristics

All geosynchronous orbits have a semi-major axis of 42,164 km (26,199 mi).[3] In fact, orbits with the same period share the same semi-major axis: $a = \sqrt[3]{\mu \left(\frac{P}{2\pi}\right)^2}$ where a=semi-major axis, P = orbital period, μ = geocentric gravitational constant.

In the special case of a geostationary orbit, the ground track of a satellite is a single point on the equator. In the general case of a geosynchronous orbit with a non-zero inclination or eccentricity, the ground track is a more or less distorted figure-eight, returning to the same places once per solar day.

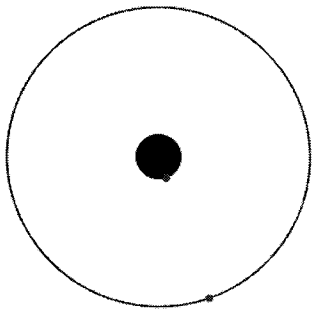
Geostationary orbit

Main article: Geostationary orbit

Geostationary orbit

From Wikipedia, the free encyclopedia

A **geostationary orbit** (or **Geostationary Earth Orbit - GEO**) is a geosynchronous orbit directly above the Earth's equator (0° latitude), with a period equal to the Earth's rotational period and an orbital eccentricity of approximately zero. These characteristics are required so that, from locations on the surface of the Earth, geostationary objects appear motionless in the sky, making the GEO an orbit of great interest to operators of communications and weather satellites. Due to the constant 0° latitude and circularity of geostationary orbits, satellites in GEO differ in location by longitude only.



Geostationary orbit

The notion of a geosynchronous satellite for communication purposes was first published in 1928 (but not widely so) by Herman Potočnik.[1] The idea of a geostationary orbit was first published on a wide scale in a paper entitled "Extra-Terrestrial Relays — Can Rocket Stations Give Worldwide Radio Coverage?" by Arthur C. Clarke, published in *Wireless World* magazine in 1945.[2] In this paper, Clarke described it as a useful orbit for communications satellites. As a result this is sometimes referred to as the **Clarke Orbit**.[3] Similarly, the **Clarke Belt** is the part of space approximately 36,000 km (22,000 mi) above sea level, in the plane of the equator, where near-geostationary orbits may be implemented. The Clarke Orbit is about 265,000 km (165,000 mi) long.

Geostationary orbits are useful because they cause a satellite to appear stationary with respect to a fixed point on the rotating Earth. As a result, an antenna can point in a fixed direction and maintain a link with the satellite. The satellite orbits in the direction of the Earth's rotation, at an altitude of 35,786 km (22,236 mi) above ground. This altitude is significant because it produces an orbital period equal to the Earth's period of rotation, known as the sidereal day.

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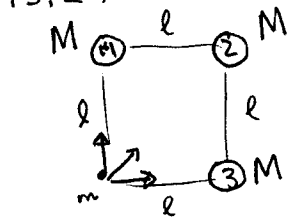
- 1 Introduction
- 2 Derivation of geostationary altitude
- 3 Practical limitations
  - 3.1 Communications
- 4 Orbit allocation
- 5 See also
- 6 Notes and references
- 7 External links

Introduction

13.21



Each mass M causes a gravitation <u>force</u> on mass "m".

Each mass M causes a gravitation <u>field</u> on mass "m".

Since the <u>force</u> and the <u>field</u> are vectors, they must be added as such.

Recall $|\vec{F}_{grav}| = G\frac{mM}{r^2} = m\left(\frac{GM}{r^2}\right) = mg$

so that $|\vec{g}| = \frac{GM}{r^2}$

#1 $|\vec{g}_1| = \frac{GM}{\ell^2}$ and $\vec{g}_1 = 0\,\hat{i} + \frac{GM}{\ell^2}\,\hat{j}$

#3 $|\vec{g}_3| = \frac{GM}{\ell^2}$ and $\vec{g}_3 = \frac{GM}{\ell^2}\,\hat{i} + 0\,\hat{j}$

#2 $|\vec{g}_2| = \frac{GM}{(\sqrt{2}\ell)^2}$ and $\vec{g}_2 = \frac{GM}{2\ell^2}\cos 45^\circ + \frac{GM}{2\ell^2}\sin 45^\circ\,\hat{j}$

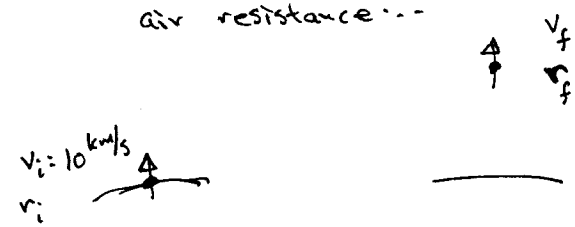
$\cos 45^\circ = \sqrt{2}/2 \qquad \sin 45^\circ = \sqrt{2}/2$

Then

$$\vec{g}_1 + \vec{g}_2 + \vec{g}_3 = \left(\frac{GM}{\ell^2} + \frac{\sqrt{2}}{4}\frac{GM}{\ell^2}\right)\hat{i} + \left(\frac{GM}{\ell^2} + \frac{\sqrt{2}}{4}\frac{GM}{\ell^2}\right)\hat{j}$$

$$= \frac{GM}{\ell^2}\left[\left(1+\frac{\sqrt{2}}{4}\right)\hat{i} + \left(1+\frac{\sqrt{2}}{4}\right)\hat{j}\right]$$

13.26 Well of course I would ignore air resistance...



$r_i = 6370$ km $= 6.370 \times 10^6$ m

Tot Init E = Tot Final E

$$\frac{1}{2}\cancel{m}v_i^2 + -G\frac{\cancel{m}M}{R} = 0 + -G\frac{\cancel{m}M}{r_f}$$

$$\frac{1}{2}(10^4)^2 + -(6.67-11)\frac{(5.96+24)}{(6.370+6)} = -(6.67-11)\frac{(5.96+24)}{r_f}$$

$r_f = 3.204 \times 10^7$ m

$= 3.204 \times 10^4$ km

$= 32040$ km

Altitude

$$\begin{array}{r} 32040 \\ -\ 6370 \\ \hline 25670 \text{ km} \end{array}$$

13.35

$m = 200\ kg$

altitude $= 200\ km$

$r = 6370 + 200$
$= 6570\ km$
$= 6.570 \times 10^6\ m$

a) $T^2 = \frac{4\pi^2}{GM} r^3$

$T^2 = \frac{4\pi^2}{(6.67\text{-}11)(5.96+24)} (6.570 \times 10^6)^3$

$T = 5304\ sec$

b) speed $V = \frac{2\pi r}{T} = \frac{2\pi (6.570 \times 10^6)}{5304}$

$= 7.779 \times 10^3\ m/s$

$= 7779\ m/s$

c) Total Energy of Satellite in orbit

$\frac{1}{2} m v^2 + -G \frac{mM}{r}$

$\frac{1}{2}(200)(7779)^2 + -(6.67\text{-}11) \frac{(200)(5.96+24)}{(6.570+6)}$

-6.050×10^9 Joules

~~the kinetic energy~~

The total energy on the surface of the Earth (including rotation) would be

$\frac{1}{2} m v^2 + -G \frac{mM}{R}$

$\frac{1}{2} 200 (463)^2 + -(6.67\text{-}11) \frac{(200)(5.96+24)}{(6.370+6)}$

-1.246×10^{10} Joules

-12.46×10^9 Joules

$V = \frac{2\pi R}{T}$

$= \frac{2\pi (6370\ km)}{86400}$

$= 463\ m/s$

the required added energy is then the difference

6.4×10^9 Joules