15.3 $x = 4\cos(3\pi t + \pi)$

a) $\omega = 3\pi$ rad/s $\quad \omega = 2\pi f$

$3\pi = 2\pi f$

$f = \frac{3}{2}$ Hz

$T = \frac{1}{f} = \frac{2}{3}$ sec

b) $A = 4$ m

c) $\phi = \pi$

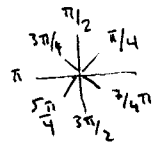
d) pos at $t = 0.25$ s

$x = 4\cos(3\pi \cdot 0.25 + \pi)$

$= 4\cos(\frac{3}{4}\pi + \pi)$

$= 4\cos(\frac{7}{4}\pi)$

$= 2.82$ m



15.2 $x = (5\text{ cm})\cos(2t + \pi/6)$

a) pos at $t=0$ $\quad x = 5\cos(2\cdot 0 + \pi/6)$

$= 4.33$ cm

b) vel at $t=0$

$v = \frac{dx}{dt} = (-)(10\ \frac{cm}{s})\sin(2t + \pi/6)$

$= -10\sin(2\cdot 0 + \pi/6)$

$= -5.0$ cm/s

c) accel at $t=0$

$a = \frac{dv}{dt} = (-)(20\ \frac{cm}{s^2})\cos(2t + \pi/6)$

$= (-)(20)\cos(2\cdot 0 + \pi/6)$

$= -17.3$ cm/s^2

d) Amplitude = 5 cm

$\omega = 2$ rad/s $\quad \omega = 2\pi f$

$2 = 2\pi f$

$f = \frac{1}{\pi} = 0.318$ Hz

$T = \frac{1}{f} = 3.14$ sec

15.6 An SHO take 12 s to go 5 complete vibs.

a) Period = $T = 12\,s / 5\,\text{oscil} = 2.4\,\text{sec}$

b) $f = \frac{1}{T} = 0.417\ \text{Hz}$

c) $\omega = 2\pi f = 2\pi\,(0.417) = 2.62\ \text{rad/s}$

15.9 Piston in engine

extremes ± 5 cm $\rightarrow A = 5$ cm

$\omega = 3600\ \frac{\text{rev}}{\text{min}} = 3600\ \frac{2\pi\ \text{rad}}{60\,s} = 377\ \text{rad/s}$

Basic eqns are

$$x(t) = A\cos(\omega t + \phi)$$
$$v(t) = -\omega A \sin(\omega t + \phi)$$
$$a(t) = -\omega^2 A \cos(\omega t + \phi)$$

max speed $= \omega A = (377)(5) = 1885\ \text{cm/s}$

max accel $= \omega^2 A = (377)^2(5) = 7.11 \times 10^5\ \text{cm/s}$

15.19 When is speed $= \frac{1}{2}$ of max speed?

$A = 3$ cm

$$x = A\cos\omega t \qquad \text{(drop the } \phi\text{)}$$
$$v = -\omega A \sin\omega t$$

want

$$v = \pm\tfrac{1}{2} v_{max} \rightarrow \pm\tfrac{1}{2}\omega A = \omega A \sin\omega t$$
$$\pm\tfrac{1}{2} = \sin\omega t$$
$$\rightarrow \omega t = \pm 30^\circ \text{ or } \pm \pi/6$$

then

$$x = A\cos\omega t = (3\,\text{cm})\cos(\pm 30^\circ) = 3\left(\tfrac{\sqrt{3}}{2}\right)\text{cm}$$

15.24 "Seconds Pendulum" $T = 2$

$$T = 2\pi\sqrt{L/g}$$

make ratio

$$\frac{\overset{2\,\text{sec}}{T_{Japan}}}{\underset{2\,\text{sec}}{T_{UK}}} = \frac{2\pi\sqrt{\frac{L_{Japan}}{g_{Japan}}}}{2\pi\sqrt{\frac{L_{UK}}{g_{UK}}}}$$

$$\sqrt{\frac{L_{UK}}{g_{UK}}} = \sqrt{\frac{L_{Japan}}{g_{Japan}}}$$

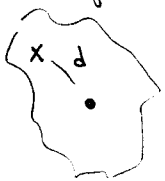
$$\frac{L_{UK}}{g_{UK}} = \frac{L_{Japan}}{g_{Japan}}$$

$$\frac{L_{UK}}{L_{Japan}} = \frac{g_{UK}}{g_{Japan}}$$

$$\frac{0.9942}{0.9927} = \frac{g_{UK}}{g_{Japan}}$$

$$1.0015 = \frac{g_{UK}}{g_{Japan}}$$

15.29 Physical Pendulum



$f = 0.450$ Hz

$M = 2.2$ kg

$d = 0.35$ m

For physical pendulum

$$I\alpha = \sum \tau = \sum r \times F$$

$$I\alpha = -Mgd \sin\theta$$

$$\frac{d^2\theta}{dt} = -\frac{Mgd}{I} \sin\theta$$

↳ ~θ for small angles

$$\omega = \sqrt{\frac{Mgd}{I}}$$

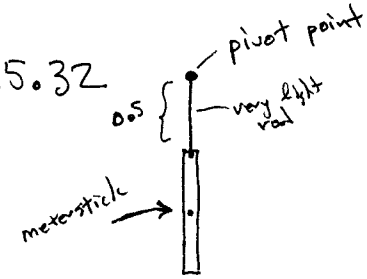
$$\omega = 2\pi f$$

$$f = \frac{1}{2\pi}\sqrt{\frac{Mgd}{I}}$$

$$0.450 = \frac{1}{2\pi}\sqrt{\frac{2.2(9.8)(0.35)}{I}}$$

$$I = 0.945 \text{ kg m}^2$$

15.32



$$I_{\substack{\text{meterstick} \\ \text{about an} \\ \text{end}}} = \frac{1}{12} M L^2 \quad (L = 1.0)$$

this formula would be true if we pivoted the meter stick about it's center. However, the true pivot point is 0.5 m away

$$I = I_{cm} + Mh^2$$

$$= \frac{1}{12}M(1)^2 + M(1)^2 = \frac{13}{12}M$$

$$\omega = \sqrt{\frac{Mgd}{I}}$$

$$\rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{\frac{13}{12}M}{M(9.8)(1)}}$$

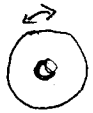
$$= 2.0880 \text{ sec}$$

Simple Pendulum of 1 m

$$T = 2\pi\sqrt{L/g} = 2\pi\sqrt{\frac{1}{9.8}} = 2.0061$$

$$\% \text{ difference} \sim \frac{2.0880 - 2.0061}{2.0061} = 4\%$$

15.33 Mechanical watch



$T = 0.25$ sec

$m = 20g = 20\times10^{-3}$ kg

$R = 0.5$ cm $= 0.5\times10^{-2}$ m

a) use formula for hollow cylinder

$$I = mR^2 = (20\times10^{-3}\,kg)(0.5\times10^{-2}\,m)^2$$
$$= 5\times10^{-8}\ kg\,m^2$$

b)

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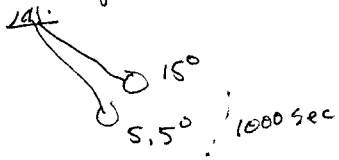
$$I\alpha = \Sigma\tau$$
$$I\alpha = -K\theta$$
$$\alpha = -\frac{K}{I}\theta$$
$$\omega^2 = \frac{K}{I}$$
$$T = 2\pi\sqrt{\frac{I}{K}}$$
$$0.25 = 2\pi\sqrt{\frac{5\times10^{-8}}{K}}$$
$$K = 3.16\times10^{-5}\ Nm/rad$$

15.35 Damped SHM – Underdamped



$$x = Ae^{-\frac{b}{2m}t}(\cos\omega t+\phi)$$

for angular motion change notation slightly

$$\theta = \underbrace{\theta_{max}\,e^{-\frac{b}{2m}t}}_{\text{Amplitude}}\cos(\omega t+\phi)$$

The amplitude changes with time (decreases)

$$5.5^\circ = 10^\circ\, e^{-\frac{b}{2m}1000}$$
$$\frac{b}{2m} = \cancel{\ } + 6.93\times10^{-4}$$

15.42 Driven SHM

$m = 0.150$ kg

$k = 6.3$ N/m

$b = 0$ – no damping

$F_0 = 1.70$ Newtons

desired amplitude $= 0.440$ m

~~Equilibrium~~ Long-Term Motion

$$x(t) = A\cos(\omega t+\phi)$$

where $A = \dfrac{F_0/m}{(\omega^2-\omega_0^2)^2+\left(\frac{b\omega}{m}\right)^2}$

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{6.3}{0.150}} = 6.48\ rad/s$$

then

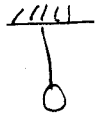
$$0.440 = \frac{1.70/0.150}{\left[\omega^2-(6.48)^2\right]^2+0^2}$$

5.075

$$\omega^2 - 41.99 = \pm 5.075$$
$$\omega^2 = 41.99 \pm 5.075$$
$$\omega = 6.860\ rad/s \text{ or } 6.076\ rad/s$$

$f = 43.08$ or 38.16 Hz

15.59 Simple Pendulum



$L = 2.23$ m

$m = 6.74$ kg

initial speed $= 2.06$ m/s when at equilibrium position.

a) Period $T = 2\pi\sqrt{L/g} = 2\pi\sqrt{\frac{2.23}{9.8}} = 3.00$ sec

b) Total Energy $= \frac{1}{2}mv_0^2 = \frac{1}{2}(6.74)(2.06)^2 = 14.3$ Joules

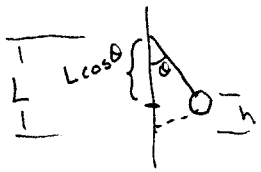
c) Max angular displacement

perhaps the most obvious way to do this is with conservation of energy

$$\frac{1}{2}\cancel{m}v_0^2 = \cancel{m}gh$$

$$\frac{1}{2}(2.06)^2 = 9.8h$$

$$h = 0.217 \text{ m}$$



$$h = L - L\cos\theta$$

$$h = L(1-\cos\theta)$$

$$0.217 = 2.23(1-\cos\theta)$$

$$\theta = 25.4^\circ$$

16.2

$\lambda = 10\ m$

$V = 1.2\ m/s$

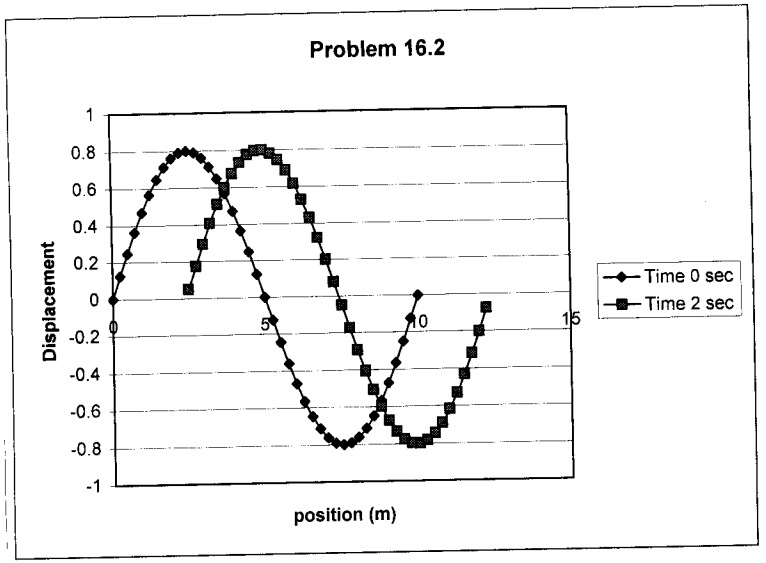
$y(x,t) = 0.8 \sin\left[0.628(x - vt)\right]$

$f\lambda = v$

$f\ 10 = 1.2$

$f = 0.12 \rightarrow T = \frac{1}{f} = 8.33\ sec$

t=0 sec t=2 sec



Only one oscillation of the wave is shown, but the basic difference is that the wave is shifted over by $(1.2\ \frac{m}{s})(2\ sec) = 2.4\ m$

16.5

$y(x,t) = 0.35 \sin\left(\underbrace{10\pi}_{\omega} t - \underbrace{3\pi}_{k} x + \frac{\pi}{4}\right)$

a) move down +x axis because there is a relative (−) sign between the t & x part of the phase

$\omega = kV$

$10\pi = 3\pi V$

$V = \frac{10}{3}\ m/s = 3.33\ m/s$

b) At $t = 0$ $x = 0.1$

$y = 0.35 \sin\left[0 - 3\pi(0.1) + \pi/4\right]$

$= 0.35 \sin\left[-0.157\right]$

$= -5.49\ cm$

c) λ f

$\lambda = \frac{2\pi}{k}$

$= \frac{2\pi}{3\pi}$

$= \frac{2}{3}\ m$

$\omega = 2\pi f$

$10\pi = 2\pi f$

$f = 5\ Hz$

d) Max transverse speed

$V_y = \frac{dy}{dt}$

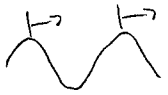
Max will be ωA

$(10\pi)(0.35)$

10.995

$11\ m/s$

16.22



$v = 196\ m/s$

$A = 0.20\ mm = 2\times10^{-4}\ m$

$f = 500\ Hz$

$\omega = 2\pi f = 3.14\times10^{3}\ rad/s$

$\mu = 4.1\ \frac{g}{m} =$

$f\lambda = v$

$500\lambda = 196$

$\lambda = 0.392\ m$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.392} = 16.02\ m^{-1}$

a) $y = A\sin[kx - \omega t]$

$= (2\times10^{-4}\ m)\sin[16x - 3140t]$

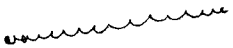
b) Using eqn from book

$v = \sqrt{T/\mu}$

$196 = \sqrt{\frac{T}{4.1\times10^{-3}}}$

$T = 157.5$ Newtons

16.23



$\mu = 5\times10^{-3}\ kg/m$

$T = 1350$ Newtons

$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{1350}{5\times10^{-3}}} = 520\ m/s$