MATH TECHNIQUES

Quadratic Eqn: $a^2x^2+bx+c=0$

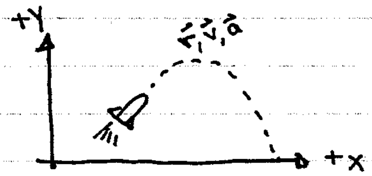
Linear Eqns $\begin{cases} 5x+4y=2 & x=? \\ 7x+2y=6 & y=? \end{cases}$

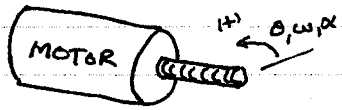
Calculus : $\frac{d}{dt}$ $\int(\cdots)\,dt$

Vectors : Magnitude, Direction, $\vec{A}$, A

$\vec{A}\cdot\vec{B}$, $\vec{A}\times\vec{B}$

MOTION = TRANSLATION + ROTATION





$\vec{r} = (x, y) = x\hat{i} + y\hat{j}$

$\vec{v} = (v_x, v_y)$ $\quad v_x = \frac{dx}{dt} \quad v_y = \frac{dy}{dt}$

$\vec{a} = (a_x, a_y)$ $\quad a_x = \frac{dv_x}{dt} \quad a_y = \frac{dv_y}{dt}$

θ

$\omega = \frac{d\theta}{dt}$

$\alpha = \frac{d\omega}{dt}$

If $a(t)$ = constant $= a$

$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

$y(t) =$

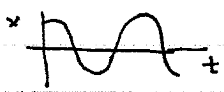
$z(t) =$

If $\alpha(t)$ = const $= \alpha$

$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

If $a(t) \propto x$ Simple Harmonic Motion

$\frac{d^2x}{dt^2} = -\omega^2 x$



$x(t) = A\cos(\omega t + \delta)$

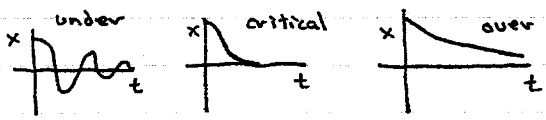
If $\alpha(t) \propto \theta$ Simple Harmonic Motio

$\frac{d^2\theta}{dt^2} = -\omega^2\theta$

$\theta(t) = \theta_0\cos(\omega t + \delta)$

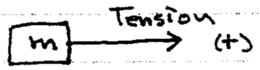
If $a(t) \propto x$ and v Damped SHM

$\frac{d^2x}{dt^2} = -\omega^2 x + \frac{b}{m}v$

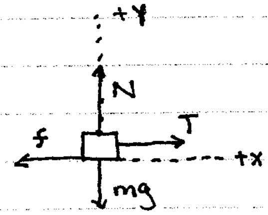


FORCES → $\vec{a}$

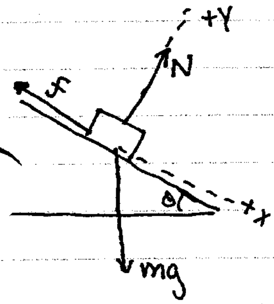
$$m\vec{a} = \sum \vec{F}\text{'s}$$



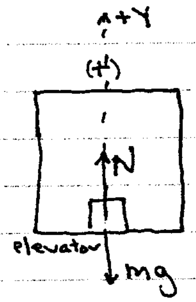
$$ma = \text{Tension}$$



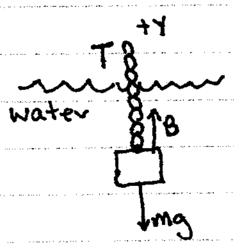
$$ma_x = T - f$$
$$= T - \mu N$$
$$ma_y = N - mg$$



$$ma_x = mg\sin\theta - f$$
$$= mg\sin\theta - \mu N$$
$$ma_y = N - mg\cos\theta$$

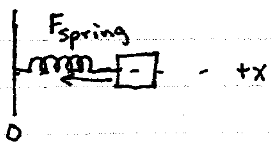


$$ma_y = N - mg$$



$$ma_y = T + B - mg$$

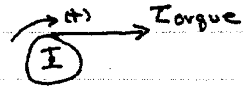
bouyant force



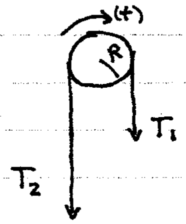
$$ma_x = -F_{spring}$$
$$= -kx$$

Torques → α

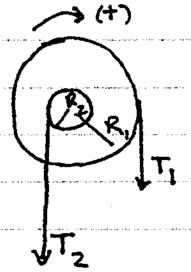
$$I\vec{\alpha} = \sum \vec{\tau}\text{'s}$$



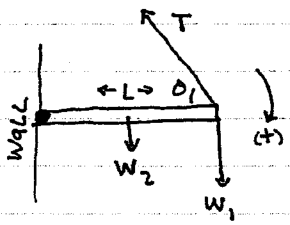
$$I\alpha = \text{Torque}$$



$$I\alpha = \tau_1 - \tau_2$$
$$= T_1R - T_2R$$



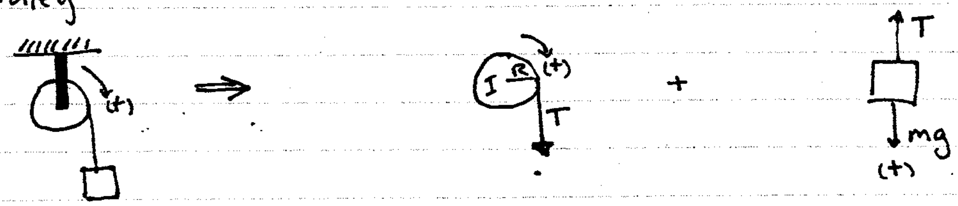
$$I\alpha = \tau_1 - \tau_2$$
$$= T_1R_1 - T_2R_2$$



$$I\alpha = W_1L + W_2(\tfrac{1}{2}L)$$
$$- T\sin\theta\, L$$

Combined Forces & Torques

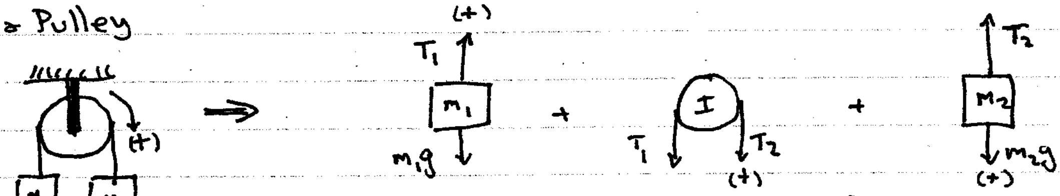
Mass & Pulley



+ constraint $a = R\alpha$

$I\alpha = TR$ $\qquad$ $ma = mg - T$

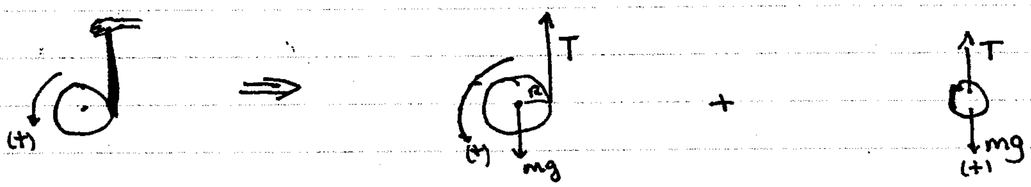
Two Masses & Pulley



+ constraint $a = R\alpha$

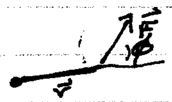
$m_1a = T_1 - m_1g$ $\qquad$ $I\alpha = T_2R - T_1R$ $\qquad$ $m_2a = m_2g - T_2$

Yo-yo



+ constraint $a = R\alpha$

$I\alpha = TR + mg(0)$
$\quad = TR$ $\qquad$ $ma = mg - T$



$\vec{\tau} = \vec{r} \times \vec{F}$

$\tau = (\text{lever arm})\, F$
$\quad = (\text{distance})\; F_T$

Other Topics:

- kinetic & static friction
- equilibrium situations $(a=0)$
- center-of-mass
- circular motion $\quad a_{radial} = \frac{v^2}{R}$
- Gravitational Force $\quad F_{grav} = -G\frac{mM}{r^2}\hat{r}$
- SHM: Resonance
- Moment of Inertia Calculations: $I = \sum m_i r_i^2 = \int r^2 dm$
- Parallel Axis Thm $\quad I = I_{cm} + Mh^2$
- Plane Figure Thm $\quad I_z = I_x + I_y$

ENERGY

(Total Initial Energy) − (frictional losses) = (total final energy)

Mechanical Energy = kinetic's + pot'l's

translational $KE = \frac{1}{2}mv^2$

rot $KE = \frac{1}{2}I\omega^2$

Grav Pot'l $U = \frac{-GmM}{r}$ or mgh

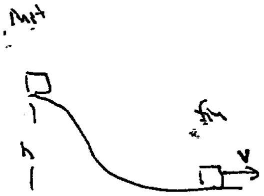
Spring $U = \frac{1}{2}kx^2$

$F(x) = -\frac{\partial U}{\partial x}$

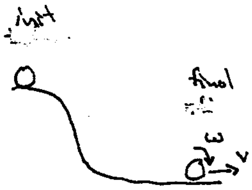
other topics: Work done by Force $= \int_i^f \vec{F}\cdot d\vec{s}$ $= \int$ comp of force along path
$= \int$ motion ~~along~~ in direction of force

work $= \int_i^f (T\cos\theta)\,dx$

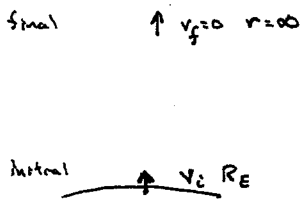
$\Delta U = U(f) - U(i) = -\int_i^f \vec{F}\cdot d\vec{s}$ for conservative forces



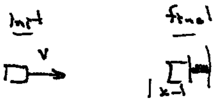
$mgh = \frac{1}{2}mv^2$



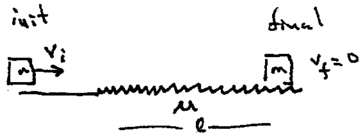
$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$



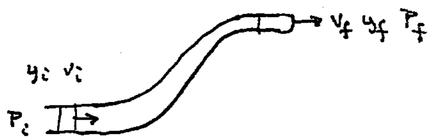
$\frac{1}{2}mv_i^2 - \frac{GmM}{R_E} = \frac{1}{2}m\,0^2 + \frac{GmM}{\infty}$
$= 0$



$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$



$\frac{1}{2}mv_i^2 - f\ell = 0$



$P_i + \frac{1}{2}\rho v_i^2 + \rho g y_i = P_f + \frac{1}{2}\rho v_f^2 + \rho g y_f$

MOMENTUM and COLLISIONS

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

$$\vec{p} = m\vec{v}$$

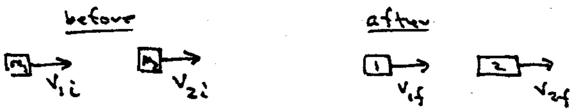
$$\frac{d\vec{L}}{dt} = \vec{\tau}_{net}$$

$\vec{L} = I\vec{\omega}$ rigid body

$= \vec{r} \times \vec{p}$ pt particles

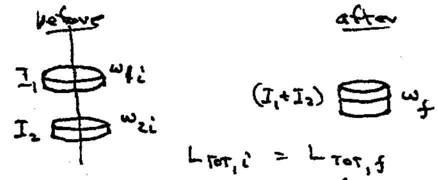
If $\vec{F}_{net} = 0$, $\vec{p} = \text{const}$

If $\vec{\tau}_{net} = 0$, $\vec{L} = \text{const.}$



$$P_{TOT\,i} = P_{TOT\,f}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$



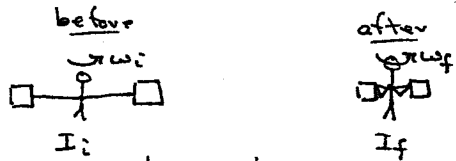
$$L_{TOT,i} = L_{TOT,f}$$

$$I_1\omega_{1i} + I_2\omega_{2i} = (I_1+I_2)\,\omega_f$$

$\vec{p}$ is vector, so $P_{xi} = P_{xf}$ $P_{yi} = P_{yf}$

$$P_{TOT\,i} = P_{TOT\,f}$$

$$m_1 v_{1i} + m_2 v_{2i} = (m_1+m_2)\,v_f$$



$$L_{TOT\,i} = L_{TOT\,f}$$

$$I_i\omega_i = I_f\omega_f$$

other topics:

cm, cm motion

coeff of restitution (rel final vel) = $-e$ (rel init vel)

procedure for collisions if don't know anything about final state

gyroscopes

Impulse = $\Delta p = F_{AVE}\,\Delta t$

If $\vec{F}_{net} \neq 0$ see previous section

If $\vec{\tau}_{net} \neq 0$ see previous section

SIMPLE HARMONIC MOTION

Differential Eqn $\quad a = \frac{d^2x}{dt^2} = -\omega^2 x$

value for ω^2 depends on the situation

mass on spring $\quad \omega^2 = k/m$
simple pendulum $\quad \omega^2 = g/L \quad$ at small θ
"physical" pendulum $\quad \omega^2 = \frac{mgd}{I} \quad$ at small θ

General solutions are of the form $\quad x(t) = A\cos(\omega t + \phi)$

Total Energy of an oscillator is $\frac{1}{2} m\omega^2 A^2$ or $\frac{1}{2} k A^2$ (mass on spring)

Period $= T = \frac{2\pi}{\omega} \qquad \omega = 2\pi f$

WAVES

Differential Eqn $\quad \frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$

Value for v^2 depends on the medium

strings $\quad v^2 = \frac{T}{\mu} \qquad$ solids $\quad v^2 = \frac{B}{\rho}$ bulk modulus

General solutions are of the form

$$y(x,t) = A\sin(kx \mp \omega t + \phi)$$

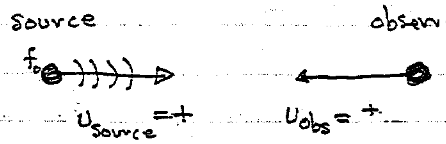
$k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f$

$+$ moves $-x$ axis
$-$ moves $+x$ axis

Fourier Decomposition - any "reasonably periodic" function can be decomposed into a basis of simple waves

$$f(x,t) = \sum_n A_n \sin(k_n x \pm \omega_n t + \phi_n)$$

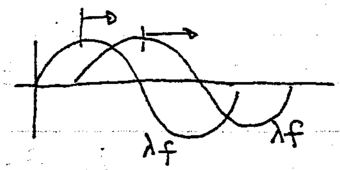
Doppler Effect



$$f_{apparent} = f_0 \left[\frac{V + u_{obs}}{V - u_{source}} \right]$$

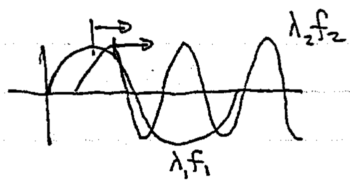
Super position of Waves

A.



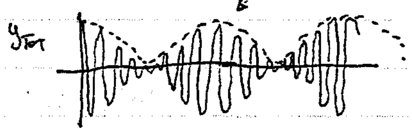
$$y_{TOT}(x,t) = 2A\cos\frac{\phi}{2}\ \sin(kx + \omega t + \phi/2)$$

B.



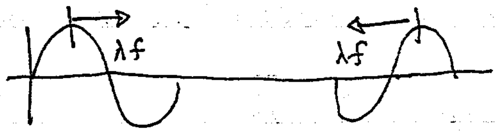
$$y_{TOT}(x,t) = 2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\sin(\bar{k}x - \bar{\omega}t)$$

envelope

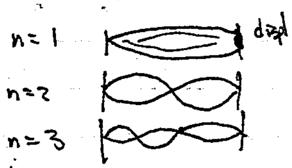
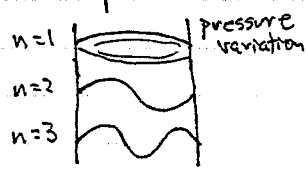
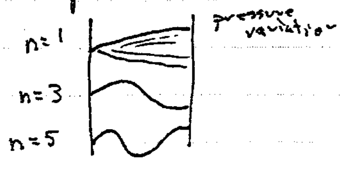


beat frequency $= \Delta f = |f_1 - f_2|$

C.



Standing Waves

Guitar String 2-fixed ends	Organ Pipe 2-open end	Organ Pipe 1-open 1-closed
		
$\lambda_1 = 2L \quad f_1 = \frac{V}{2L}$	$\lambda_1 = 2L \quad f_1 = \frac{V}{2L}$	$\lambda_1 = 4L \quad f_1 = \frac{V}{4L}$
$\lambda_n = \frac{\lambda_1}{n} \quad f_n = nf_1$	$\lambda_n = \frac{\lambda_1}{n} \quad f_n = nf_1$	$\lambda_n = \frac{\lambda_1}{n} \quad f_n = nf_1$
		even harmonics missing