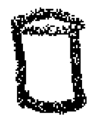
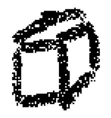
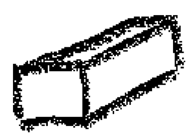
# VOLUMES AND SURFACE AREAS

1. Fill in the following table listing formulae for calculating volumes and surface areas

| Object | Volume | Surface Area |
|---|---|---|
| Spherical ball of radius *R* | $\frac{4}{3}\pi R^3$ | $4\pi R^2$ |
| Cylinder of radius *r* and height *h* | $\pi r^2 h$ | $2\pi r h + 2\pi r^2$ |
| Cube with sides of length *a* | $a^3$ | $6a^2$ |
| Rectangular solid with sides of length *l w h* | $lwh$ | $2lw + 2wh + 2hl$ |







# COUPLED EQUATIONS

1. There are those situations where there are several problems and you have to find a common solution agreeable to both:

Consider the two equations

$$2x + y = 1 \quad \text{and} \quad 3x + 4y = 9$$

The x,y solution is found by substituting one equation into the other. Do it.

$y = 1 - 2x \rightarrow 3x + 4(1-2x) = 9$

$3x + 4 - 8x = 9$

$-5x = 5$

$x = -1$

$y = 1 - 2(-1)$

$y = 3$

$\boxed{x = -1 \quad y = 3}$

2. Find a single common x,y,z solution to the two equations: $5x + 2y + z = 0$ and $3x + y + 2z = 1$

No single unique solution possible.

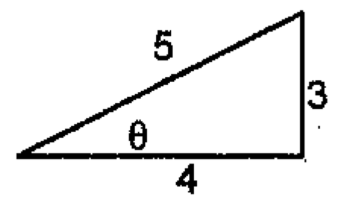
3 unknowns requires 3 equations and we were only given 2 eqns.

# Practice with Trig Functions

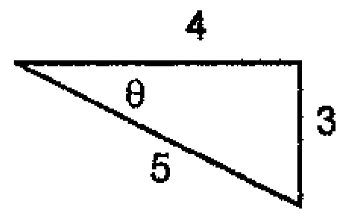
USMC and USN officers find themselves in all-sorts of orientations. So do their right triangles.

Give values for cos θ , sin θ , and tan θ in each of the following situations. The lengths of the sides are indicated. Leave numbers in rational form (i.e. 3/5 vs 0.6)

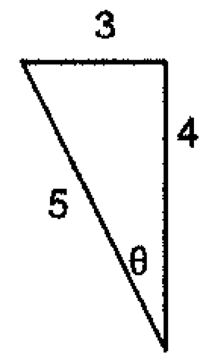
on your belly



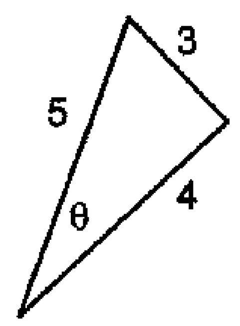

on your back

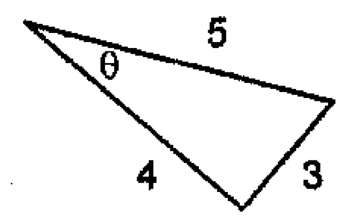


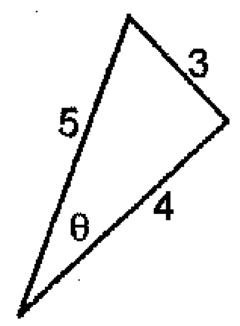

up a tree



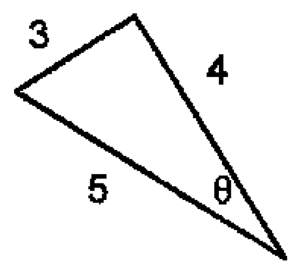

.................................SWO in a storm surge..............................



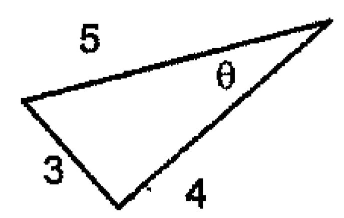



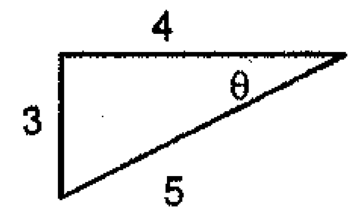

going ballistic




headn' for the deck




inverted backards




All are

$\cos\theta = \frac{4}{5} \qquad \sin\theta = \frac{3}{5}$

# VECTOR PRACTICE

1. Let $\vec{V} = 2.00\hat{i} + 6.00\hat{j} - 3.00\hat{k}$. The magnitude of $\vec{V}$ is:
   A) 5.00
   B) 5.57
   C) 7.00
   D) 7.42
   E) 8.54

$|\vec{v}| = \sqrt{2^2 + 6^2 + (-3)^2}$
4 36 9
$= 7$

2. Let $\vec{A} = (2\text{ m})\hat{i} + (6\text{ m})\hat{j} - (3\text{ m})\hat{k}$ and $\vec{B} = (4\text{ m})\hat{i} + (2\text{ m})\hat{j} + (1\text{ m})\hat{k}$. The vector sum $\vec{S} = \vec{A} + \vec{B}$ is:
   A) $(6\text{ m})\hat{i} + (8\text{ m})\hat{j} - (2\text{ m})\hat{k}$
   B) $(-2\text{ m})\hat{i} + (4\text{ m})\hat{j} - (4\text{ m})\hat{k}$
   C) $(2\text{ m})\hat{i} - (4\text{ m})\hat{j} + (4\text{ m})\hat{k}$
   D) $(8\text{ m})\hat{i} + (12\text{ m})\hat{j} - (3\text{ m})\hat{k}$
   E) none of these

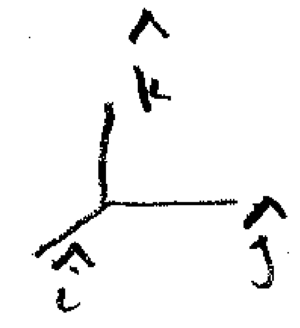
Add like components

3. Let $\vec{A} = (2\text{ m})\hat{i} + (6\text{ m})\hat{j} - (3\text{ m})\hat{k}$ and $\vec{B} = (4\text{ m})\hat{i} + (2\text{ m})\hat{j} + (1\text{ m})\hat{k}$. Then $\vec{A} \cdot \vec{B}$ equals:
   A) $(8\text{ m})\hat{i} + (12\text{ m})\hat{j} - (3\text{ m})\hat{k}$
   B) $(12\text{ m})\hat{i} - (14\text{ m})\hat{j} - (20\text{ m})\hat{k}$
   C) 23
   D) 17
   E) none of these

$2 \cdot 4 + 6 \cdot 2 + (-3)1$
8 12 $-3$

4. Vectors $\vec{A}$ and $\vec{B}$ each have magnitude $L$. When drawn with their tails at the same point, the angle between them is 30°. The magnitude of $\vec{A} \times \vec{B}$ is:
   A) $L^2/2$
   B) $L^2$
   C) $\sqrt{3}L^2/2$
   D) $2L^2$
   E) none of these

$|\vec{A} \times \vec{B}| = AB\sin\theta$
$= L \cdot L \sin 30^\circ$
$= L^2/2$




5. The value of $\hat{i} \cdot (\hat{j} \times \hat{k})$ is:
   A) zero
   B) +1
   C) −1
   D) 3
   E) $\sqrt{3}$

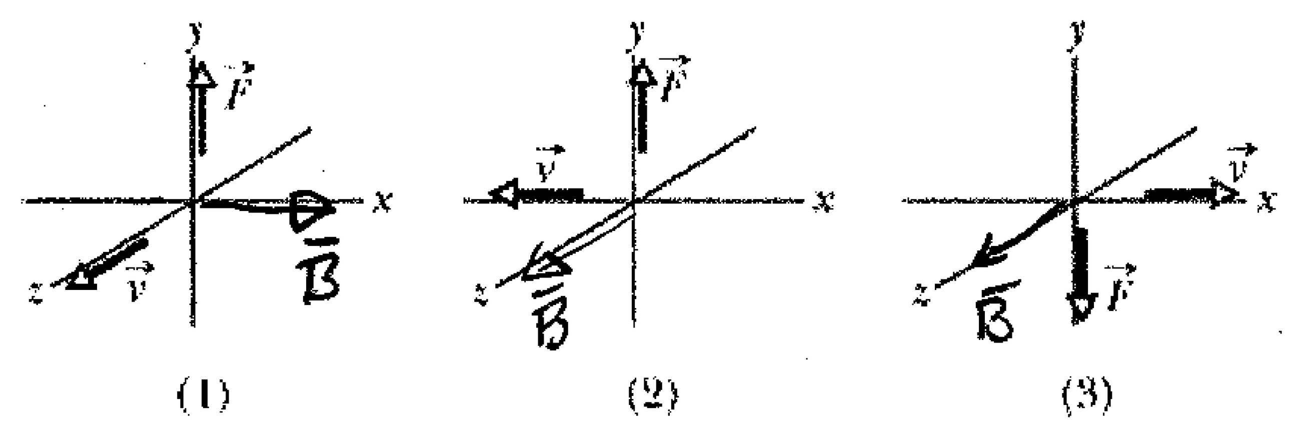
$\hat{j} \times \hat{k} = \hat{i}$
$\hat{i} \cdot \hat{i} = 1$

6. The value of $\hat{k} \cdot (\hat{k} \times \hat{i})$ is:
   A) zero
   B) +1
   C) −1
   D) 3
   E) $\sqrt{3}$

the cross product generates a vector $\perp$ to the original vectors

7. In the formula $\vec{F} = q\vec{v} \times \vec{B}$:
   A) $\vec{F}$ must be perpendicular to $\vec{v}$ but not necessarily to $\vec{B}$
   B) $\vec{F}$ must be perpendicular to $\vec{B}$ but not necessarily to $\vec{v}$
   C) $\vec{v}$ must be perpendicular to $\vec{B}$ but not necessarily to $\vec{F}$
   D) all three vectors must be mutually perpendicular
   E) $\vec{F}$ must be perpendicular to both $\vec{v}$ and $\vec{B}$

8. If given the vector formula $\vec{F} = q\,(\vec{v} \times \vec{B})$ and $\vec{v}$ is perpendicular to $\vec{F}$ as shown in each of the figures below, then what is the direction of $\vec{B}$ in each of the three situations when constant $q$ is positive?




Answers: (1) $+\hat{i}$ (2) $+\hat{k}$ (3) $+\hat{k}$

# INTEGRAL PRACTICE

Using a suitable reference, such as provided hints, the textbook front inside cover, the textbook Appendix E, or your calculator, perform the following integrals

$$\int_{-60^\circ}^{+60^\circ} \cos\theta \, d\theta = \sin\theta \Big|_{-60}^{60} = \sin 60^\circ - \sin(-60^\circ)$$
$$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$$
$$= \sqrt{3}$$

$$\int_{\infty}^{R} \frac{1}{r^2} dr = -\frac{1}{r}\Big|_{\infty}^{R} = -\frac{1}{R} - -\frac{1}{\infty} = -\frac{1}{R}$$

$$\int_{a}^{b} \frac{1}{r} dr = \ln r \Big|_{a}^{b} = \ln b - \ln a = \ln(b/a)$$

$$\int_{0}^{L} \frac{2x}{\left(x^2+d^2\right)^{1/2}} dx$$

Hint: Let $u = x^2 + d^2$

$du = 2x\,dx$

$$\int \frac{du}{u^{1/2}}$$
$$\int u^{-1/2} du$$
$$2u^{1/2}\Big]$$
$$2\left(x^2+d^2\right)^{1/2}\Big]_0^L = 2\sqrt{L^2+d^2} - 2d$$

$$\int_{0}^{R} \frac{1}{\left(z^2+r^2\right)^{1/2}} dr$$

The textbook inside front cover and Appendix E give

$$\ln\left(r+\sqrt{r^2+z^2}\right)\Big|_0^R = \ln\left(R+\sqrt{R^2+z^2}\right) - \ln z$$

$$\int_{0}^{R} \frac{2r}{\left(z^2+r^2\right)^{3/2}} dr$$

Hint: let $u = z^2 + r^2$

$du = 2r\,dr$

$$\int \frac{du}{u^{3/2}}$$
$$\int u^{-3/2} du = \frac{1}{-1/2} u^{-1/2}\Big] = -2u^{-1/2}\Big] = \frac{-2}{\sqrt{z^2+r^2}}\Bigg|_0^R$$

$$\int_{0}^{L} \frac{1}{\left(x^2+d^2\right)^{3/2}} dx$$

The textbook inside front cover and Appendix E give

$$\frac{x}{d^2\left(x^2+d^2\right)^{1/2}}\Bigg]_0^L = \frac{L}{d^2\left(L^2+d^2\right)^{1/2}} - 0$$
$$= \frac{L}{d^2\left(L^2+d^2\right)^{1/2}}$$