23-23

cylindrical symmetry
∴ choose gaussian surface as cylinder

a) $\oint \bar{E} \cdot d\bar{A} = \frac{Q_{encl}}{\epsilon_0}$

$E \cdot 2\pi r L = \frac{Q}{\epsilon_0}$

$E = \frac{1}{2\pi\epsilon_0} \frac{Q/L}{r}$

$E = 2.3\times10^5$ N/C
$r = 0.12$ m
$L = 0.42$ m

$2.3\times10^5 = \frac{1}{2\pi(8.85-12)} \frac{Q/0.42}{0.12}$

$Q = 6.44\times10^{-7}$ Coul

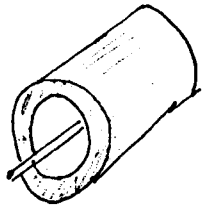
b) Redo with $E = 2.3\times10^5$ N/C
$r = 0.08$ m
$L = 0.28$ m

$E = \frac{1}{2\pi\epsilon_0} \frac{Q/L}{r}$

$2.3\times10^5 = \frac{1}{2\pi(8.85-12)} \frac{Q/0.28}{0.08}$

$Q = 2.86\times10^{-7}$ Coul

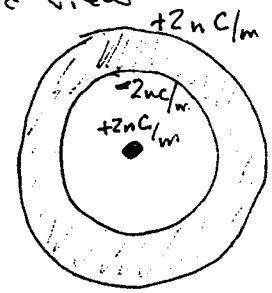
23-28



central wire
$\lambda = 2.0 \times 10^{-9}$ C/m

thick cyl shell
$r_{inner} = 5$ cm
$r_{outer} = 10$ cm
Net charge = 0
conductor

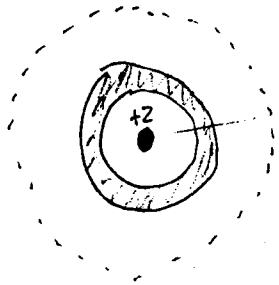
End View



charge density
on inner surf $\lambda = -2$ nC/m
on outer surf $\lambda = +2$ nC/m
to get areal charge density nC/m^2, divide by the circumference $2\pi r$

Find E-field outside at $r = 15$ cm

END VIEW



$$\oint \bar{E} \cdot d\bar{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$E\, 2\pi r L = \frac{Q_{encl}}{\epsilon_0}$$

$$E = \frac{1}{2\pi\epsilon_0} \frac{Q_{encl}/L}{r}$$

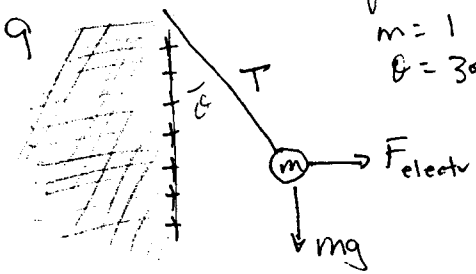
$$= \frac{1}{2\pi\epsilon_0} \frac{\lambda_{encl}}{r}$$

$$= \frac{1}{2\pi(8.85\text{-}12)} \frac{(2\times10^{-9}\ \text{C/m})}{0.15}$$

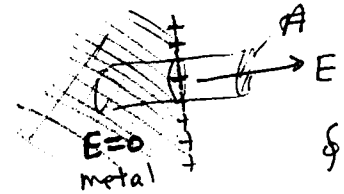
$$= 239.9\ \text{N/C}$$

23-39

$q = 2\times10^{-8}$ Coul
$m = 1$ mg
$\theta = 30°$



E-field at surf of conductor

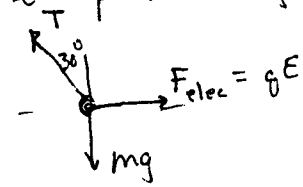


$$\oint E \cdot dA = \frac{Q_{encl}}{\epsilon_0}$$

$$EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \sigma/\epsilon_0$$

Then set up free body diagram



$$m\cancel{a_x} = \Sigma F_x \qquad m\cancel{a_y} = \Sigma F_y$$

$$0 = qE - T\sin 30° \qquad 0 = T\cos 30° - mg$$

$$T\sin 30° = qE \qquad T\cos 30° = mg$$

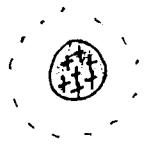
divide

$$\frac{T\sin 30°}{T\cos 30°} = \frac{qE}{mg} = \frac{q\,\sigma/\epsilon_0}{mg}$$

$$\tan 30° = \frac{(2\times10^{-8})\ \sigma/(8.85\text{-}12)}{(1\text{-}3)\ 9.8}$$

$$\sigma = 2.50\times10^{-6}\ \text{Coul/m}^2$$

23-44 Solid Insulating Ball



"Outside" the sphere

$$\oint E \cdot dA = \frac{Q_{encl}}{\epsilon_0}$$

$$E\, 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

This "outside" formula is good at the surface. From the graph in the book, we see

E @ surface $= 5 \times 10^7$ N/C @ 2 cm

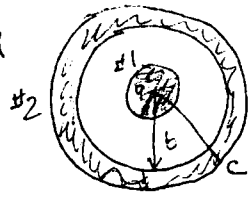
so that

$$5 \times 10^7 = \frac{1}{4\pi(8.85-12)} \frac{Q}{0.02}$$

$$Q = 1.11 \times 10^{-4} \text{ Coul}$$

is the total charge distributed throughout the sphere.

23-49



The problem separates into four regions

$r<a$

$a<r<b$

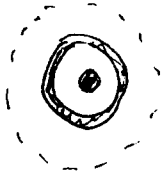
$b<r<c$

$c<r$

$q_1 = 5\ fC = 5\times10^{-15}\ C$

$q_2 = -5\ fC = -5\times10^{-15}\ C$

Outside everything $c<r$



$$\oint \bar{E}\cdot d\bar{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$E\cdot 4\pi r^2 = \frac{Q_{encl}}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q_{encl}}{r}$$

where $Q_{encl} = 0$ total

inside shell $b<r<c$

The shell is a conductor, so that $E=0$

middle region $a<r<b$

$$\oint \bar{E}\cdot d\bar{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$E\ 4\pi r^2 = \frac{Q_{encl}}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q_{encl}}{\epsilon_0}$$

where $Q_{enc} = 5\times10^{-15}\ C$

inside central ball $r<a$

The gaussian sphere encloses only a fraction of the charge $+5\times10^{-15}\ C$.

The fraction is given by the ratio of the volumes $\frac{4}{3}\pi r^3$ to $\frac{4}{3}\pi a^3$

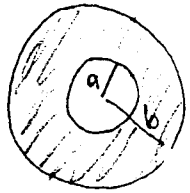
$$\oint \bar{E}\cdot d\bar{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$E\ 4\pi r^2 = \frac{q_1 \frac{r^3}{a^3}}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0}\ q_1\ \frac{r}{a^3}$$

then plug in the numbers

23-52



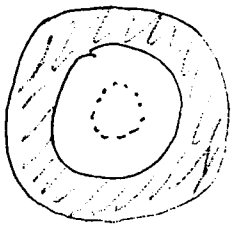
The problem separates into three regions

$r < a$

$a < r < b$

$b < r$

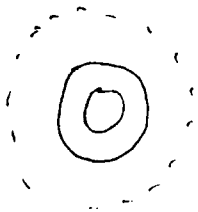
Inside $r < a$



for any gaussian surface drawn inside the shell, it will enclose no charge

so $E = 0$

Outside $b < r$



the gaussian surface will enclose all the charge

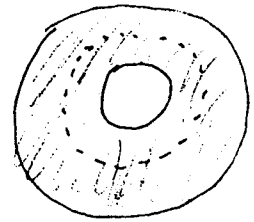
$Q_{encl} = \rho \cdot Vol$

where $V = \underbrace{\frac{4}{3}\pi b^3}_{\text{ball}} - \underbrace{\frac{4}{3}\pi a^3}_{\text{hole}}$

$$\oint \bar{E} \cdot d\bar{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$E \; 4\pi r^2 = \frac{\rho\left(\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3\right)}{\epsilon_0}$$

Middle Region $a < r < b$



The gaussian surface will enclose a portion of the charge

$Q_{encl} = \rho \cdot Vol$

where $V = \frac{4}{3}\pi r^3 - \underbrace{\frac{4}{3}\pi a^3}_{\text{hole}}$

Then

$$\oint \bar{E} \cdot d\bar{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$E \; 4\pi r^2 = \frac{\rho\left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3\right)}{\epsilon_0}$$

then plug numbers in.