2-65

determine speeds

Electron $KE = 9\ GeV = 9000\ MeV$

$KE = (\gamma - 1)mc^2$

electron $mc^2 = 0.511\ MeV$

$9000 = (\gamma - 1)0.511$

$\gamma = 1.76 \times 10^4$

$\gamma^2 = 3.10 \times 10^8$

$\frac{1}{1-\beta^2} = 3.10 \times 10^8$

$\beta = 0.999999998$

Positron $KE = 3.1\ GeV = 3100\ MeV$

$KE = (\gamma - 1)mc^2$

positron $mc^2 = 0.511\ MeV$

$3100 = (\gamma - 1)\ 0.511$

$\gamma = 6068$

$\gamma^2 = 3.68 \times 10^7$

$\frac{1}{1-\beta^2} = 3.68 \times 10^7$

$\beta = 0.999999986$

2-66 Tevatron

$KE = (\gamma - 1)mc^2$

proton $mc^2 = 938.27\ MeV$ (front cover)

This works the same as last problem inside

	KE (MeV)	γ	β
Cockroft-Walton	0.750	1.0008	0.056
Linac	400	1.43	0.870
Booster	8000	9.53	0.99994
Main Ring	150000	161	0.999999993
Tevatron	1,000,000	1067	1.000000000

2-76 Calc energy needed to accelerate spaceship of $m = 10^4$ kg to $v = 0.3c$

Compare to annual Earth energy usage of 10^{21} J

Required work = change in KE

$$KE = (\gamma - 1)mc^2$$

$$\beta = 0.3$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = 1.048$$

$$= (1.048 - 1)(10^4 \text{ kg})(3 \times 10^8 \text{ m/s})^2$$

$$= 4.35 \times 10^{19} \text{ Joules}$$

which is 16 days of Earth energy generation.

2-83 Muon $mc^2 = 106$ MeV

$KE = 200$ MeV

Calculate a) speed, b) momentum, a) total energy

a) $E_{TOT} = KE + mc^2 = 200 + 106 = 306$ MeV

b) $E_{TOT}^2 - p^2c^2 = m^2c^4$

$$(306)^2 - p^2c^2 = (106)^2$$

$$pc = 287 \text{ MeV}$$

$$p = 287 \text{ MeV/c}$$

c) $KE = (\gamma - 1)mc^2$

$$200 = (\gamma - 1)\,106$$

$$\gamma = 2.89$$

$$\frac{1}{1-\beta^2} = 2.89$$

$$\beta = 0.81$$

2-85 Deuterium-Tritium Reaction

$$^2H + {}^3H \longrightarrow n + {}^4He$$

$m_{^2H} = 2.014102$ amu

$m_{^3H} = 3.016049$ amu

$m_n = 1.008665$

$m_{^4He} = 4.002603$

use $c^2 = 931.5\ \text{MeV/amu}$

$$\text{Energy Released} = \left[\left(\begin{matrix}\text{Mass}\\ \text{reactants}\end{matrix}\right) - \left(\begin{matrix}\text{mass}\\ \text{products}\end{matrix}\right)\right] c^2$$

$$= \left[(2.014102 + 3.016049) - (1.008665 + 4.002603)\right] c^2$$

$$= 17.6\ \text{MeV}$$

2-98 Blackbird SR-71

~ max speed 2200 mph = 983 m/s

length 32.74 m

a) by what percentage would it appear contracted?

$$\text{Contraction} = \frac{L_0}{\gamma} - L_0 = L_0\left(\frac{1}{\gamma} - 1\right)$$

So we want

$$\frac{1}{\gamma} - 1 = \left(\frac{1}{\sqrt{1-\beta^2}}\right)^{-1} - 1$$

$$\beta = \frac{v}{c} = \frac{983}{3\times10^8} = 3.3\times10^{-6}$$

$$= (1-\beta^2)^{1/2} - 1$$

$$\approx -\tfrac{1}{2}\beta^2 \approx 5.4\times10^{-12}$$

or 5.4×10^{-10} %

b) Time Difference

$$\text{Diff} = \gamma T_0 - T_0 = [\gamma - 1]T_0$$

$$= \left[(1-\beta^2)^{-1/2} - 1\right] T_0$$

$$\approx +\tfrac{1}{2}\beta^2 T_0$$

$$\text{estimate } T_0 = \frac{3200\times10^3}{983}$$

$$= 3255\ \text{sec}$$

$$\approx \tfrac{1}{2}(3.3\times10^{-6})^2 (3255\ \text{sec})$$

$$\approx 1.77\times10^{-8}\ \text{sec}$$

$$\approx 18\ \text{nsec}$$