5-3 Potassium Chloride

lattice spacing 0.314 nm

First Bragg peak at 12°

$n\lambda = 2d \sin\theta$

so interpret what we are told that

$2\theta = 12°$

$\theta = 6°$

$1\lambda = 2\ 0.314 \sin 6°$

$\lambda = 6.56 \times 10^{-2}$ nm

$= 6.56 \times 10^{-11}$ m

X-Ray energy

for photons

$E_\gamma = hf = \frac{hc}{\lambda}$

$= \frac{(6.626\text{-}34)(3\text{+}8)}{6.56 \times 10^{-11}}$

$= 3.03 \times 10^{-15}$ Joules

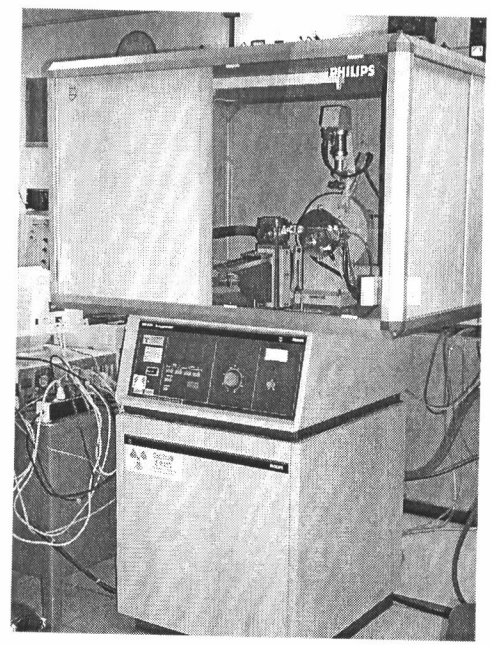
$= 1.89 \times 10^{4}$ eV

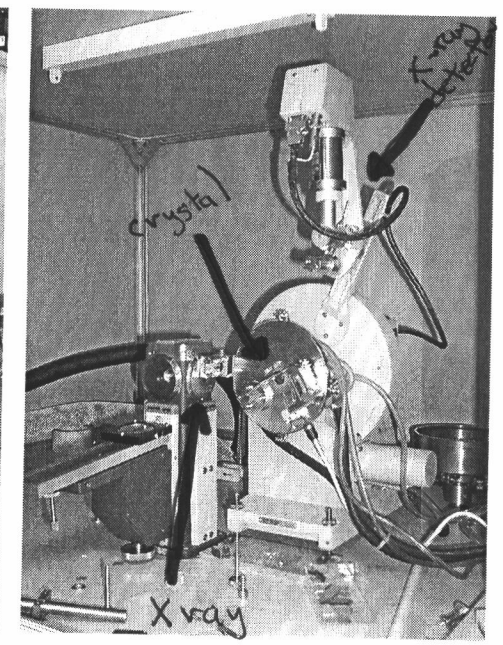
$= 18.9$ keV

what other angles do bright peaks occur?

$n\lambda = 2d \sin\theta$

n	θ	2θ
1	6°	12°
2	12.1°	24.1°
3	18.3°	36.5°
4	24.7°	49.4°
5	31.5°	62.9°
6	38.8°	77.6°
~~7~~	~~46.9°~~	~~93.9°~~





5-62

$d = 0.156\,nm$

$2\theta = 26^\circ \rightarrow \theta = 13^\circ$

$n\lambda = 2d\sin\theta$

$1\lambda = 2\ 0.156 \sin 13^\circ$

$\lambda = 7.02\times10^{-2}\ nm$

Then if neutrons are non-relativistic

$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

$p = \frac{h}{\lambda} = \frac{6.626\text{-}34\ Js}{7.02\text{-}11\ m}$

$= 9.44\times10^{-24}$

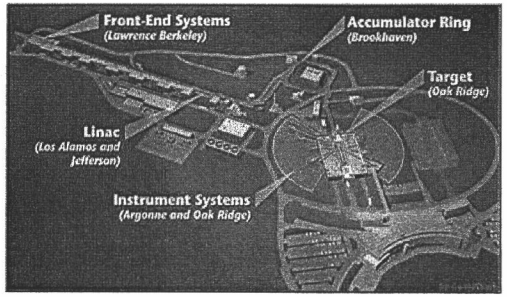
$m = 1.67\times10^{-27}\ kg$

$KE = \frac{(9.44\times10^{-24})^2}{2(1.67\times10^{-27})}$

$KE = 2.67\times10^{-20}$ Joules

$KE = 0.167\ eV$

How the Spallation Neutron Source Works



Conceptual drawing of the SNS site.

Negatively charged hydrogen ions are produced by an ion source. Each ion consists of a proton orbited by two electrons. The ions are injected into a linear accelerator, which accelerates them to very high energies. The ions are passed through a foil, which strips off each ion's two electrons, converting it to a proton. The protons pass into a ring where they accumulate in bunches. Each bunch of protons is released from the ring as a pulse. The high-energy proton pulses strike a heavy-metal target, which is a container of liquid mercury. Corresponding pulses of neutrons freed by the spallation process will be slowed down in a moderator and guided through beam lines to areas containing special instruments such as neutron detectors. Once there, neutrons of different energies can be used in a wide variety of experiments.

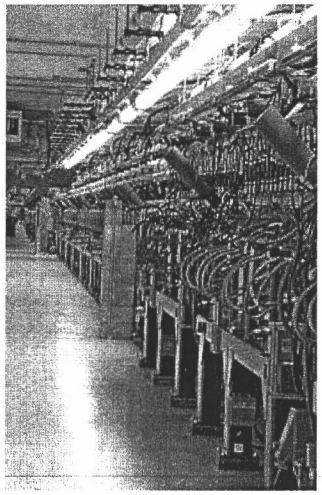
The baseline design calls for an accelerator system consisting of an ion source, full-energy linear accelerator (linac), and an accumulator ring that combine to produce short, powerful pulses of protons. These proton pulses impinge onto a mercury target to produce neutrons through the spallation nuclear reaction process. At full power, SNS will deliver 1.4 million watts (1.4 MW) of beam power onto the target, and it has been designed with the flexibility to provide additional scientific output in the future. This approach is intended to provide a facility that will meet the neutron intensity needs of the science community well into the next century.

Ion Source

The SNS front-end system includes an ion source, beam formation and control hardware, and low-energy beam transport and acceleration systems. The ion source produces negative hydrogen (H-) ions—hydrogen with an additional electron attached—that are formed into a pulsed beam and accelerated to an energy of 2.5 million electron volts (MeV). This beam is delivered to a large linear accelerator (linac).

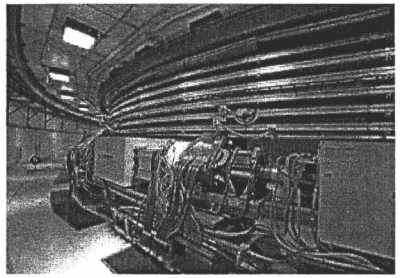
Linac

The linac accelerates the H- beam from 2.5 to 1000 MeV, or 1 GeV. The linac is a superposition of normal conducting and superconducting radio-frequency cavities that accelerate the beam and a magnetic lattice that provides focusing and steering. Three different types of accelerators are used. The first two, the drift-tube linac and the coupled-cavity linac, are made of copper, operate at room temperature, and accelerate the beam to about 200 MeV. The remainder of the acceleration is accomplished by superconducting niobium cavities. These cavities are cooled with liquid helium to an operating temperature of 2 K. Diagnostic elements provide information about the beam current, shape, and timing, as well as other information necessary to ensure that the beam is suitable for injection into the accumulator ring and to allow the high-power beam to be controlled safely.



The 1,000-foot SNS linear accelerator is made up of three different types of accelerators. It is the first of its kind used to generate a pulsed energy beam.

Accumulator Ring

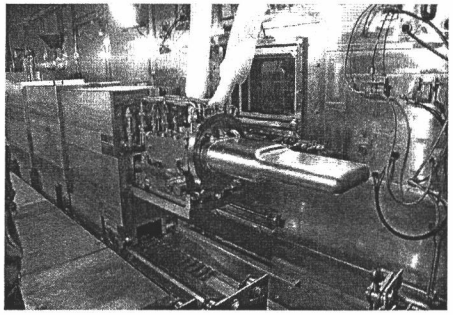


The SNS ring intensifies the high-speed ion beam and shoots it at the mercury target 60 times a second.

The accumulator ring structure bunches and intensifies the ion beam for delivery onto the mercury target to produce the pulsed neutron beams. The intense H- beam from the linac must be sharpened more than 1000 times to produce the extremely short, sharp bunch of neutrons needed for optimal neutron-scattering research. To accomplish this goal, the H- pulse from the linac is wrapped into the ring through a stripper foil that strips the electrons from the negatively charged hydrogen ions to produce the protons (H+) that circulate in the ring. Approximately 1200 turns are accumulated, and then all these protons are kicked out at once, producing a pulse less than 1 millionth of a second (10-6seconds) in duration that is delivered to the target. In this way, short, intense proton pulses are produced, stored, and extracted at a rate of 60 times a second to bombard the target.

Target

Because of the enormous amount of energy that the short, powerful pulses of the incoming 1-GeV proton beam will deposit in the spallation target, it was decided to use a liquid mercury target rather than a solid target such as tantalum or tungsten. SNS will be the first scientific facility to use pure mercury as a target for a proton beam.



Mercury was chosen for the target for several reasons: (1) it is not damaged by radiation, as are solids; (2) it has a high atomic number, making it a source of numerous neutrons (the average mercury nucleus has 120 neutrons and 80 protons); and (3), because it is liquid at room temperature, it is better able than a solid target to dissipate the large, rapid rise in temperature and withstand the shock effects arising from the rapid high-energy pulses.

The neutrons coming out of the target must be turned into low-energy neutrons suitable for

The curved, rectangular object is the SNS target. Inside is liquid mercury, where spallation takes place.

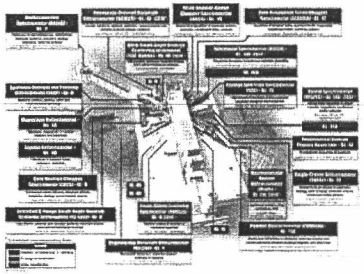
research—that is, they must be moderated to room temperature or colder. The neutrons emerging from the target are slowed down by passing them through cells filled with water (to produce room-temperature neutrons) or through containers of liquid hydrogen at a temperature of 20 K (to produce cold neutrons). These moderators are located above and below the target. Cold neutrons are especially useful for research on polymers and proteins.

SNS is an inherently safe way to produce neutrons because the neutron production stops when the proton beam is turned off. It also produces few hazardous materials. To maximize the safety of the facility, the SNS is designed to have many levels of containment to keep potentially hazardous material from getting into the environment.

Instrumentation and Experiment Facilities

SNS will initially have one target station operating at a frequency of 60 Hertz (Hz). Two "thermal" moderators and two "cold" moderators will be used to service 18 beam lines, and a variety of instruments will be constructed on these beam lines. For the experiment facilities, the SNS expects 1000 to 2000 users each year from all walks of science and industry. Because not all these users will be experts in neutron scattering, the SNS will provide scientists and technicians to maintain and operate the instruments and work closely with the user community.

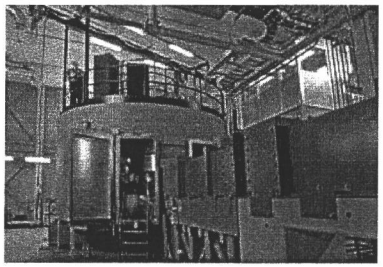
The broad user community has been and will continue to be involved in the selection, design, construction, and operation of the instruments. The user community recommended and prioritized a suite of ten instruments for initial installation at the 60-Hz target station. Eight beam lines have been be reserved for cooperative research teams to develop and install additional instruments.



Schematic SNS instrument suite for showing the 18 currently planned beam lines. Final instrumentation will be determined by the user community through the SNS Instrument Oversight Committee. (Click to see a detailed view of the instruments for each beam line and the scientific areas where each instrument will be.)

Scientific Research at SNS

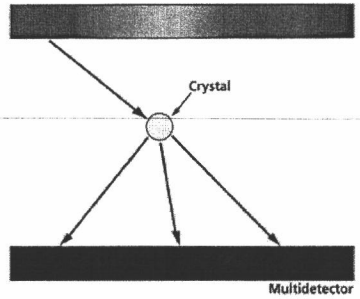
The instruments at the SNS, such as neutron spectrometers, will be used to determine the positions, or arrangements, of atoms in crystals, ceramics, superconductors, and proteins. How does a neutron spectrometer work? A pulse of neutrons generated by the spallation source follows a flight path to the sample. Because the neutrons have varying energies and wavelengths, they spread out in time, presenting a continuous spectra to the sample. When the distance between atoms in a crystal matches the wavelength of an incident neutron, that neutron is scattered into a



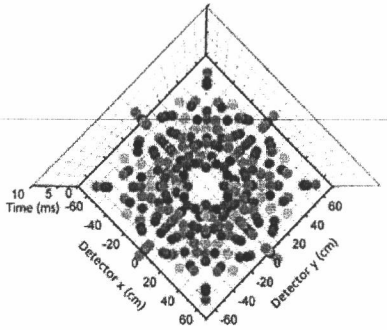
multidetector that records the position (scattering angle) and time of arrival of the scattered neutron. The result is a pattern of peaks showing the different positions and arrival times of various numbers of neutrons reaching each point in the multidetector. This pattern tells scientists how different atoms are arranged in the crystal.

Exterior of the first completed SNS instrument, the Backscattering Spectrometer.

Instrumentation based on the same principles can be used to determine the atomic structure of glasses and complex fluids or the residual stresses in industrial parts. Instruments to measure inelastic scattering will require measurement of the time of neutron travel over paths leading to and from the sample. In this way, instruments can determine the excitation spectra of materials of importance and thus the nature of the forces that hold the atoms in place. The time-of-flight technique makes it possible to collect a large number of data points for each neutron pulse. The efficiency of instruments that measure neutron time of flight and the ability of accelerator-based spallation neutron sources to produce pulsed beams of increasing intensity promise to provide continuously improved neutron sources in the future.



Atomic structures are obtained by letting the full neutron spectrum from the source scatter from a crystal into a position-sensitive multidetector.



Pattern obtained from the multidetector in time and space for a crystal of the high-temperature superconductor $YBa_2Cu_3O_7$. The position of the atoms can be obtained from the pattern.

5-5 Rock, thrown

$m = 3\,kg$

$v = 6\,m/s$

$p = mv = 3.6 = 18\ kg\,m/s$

$$\lambda = \frac{h}{p} = \frac{6.626^{-34}}{18} = 3.68 \times 10^{-35}\,m$$

which is what you call small.

Because the wavelength is so much smaller than any feature we encounter with water, it is best to describe the rock with a <u>particle model</u>

5-7 Transmission electron microscope.

- Assume electrons are non-relativistic because the KE in this problem are $\ll mc^2 = 0.511\,MeV$

$KE = 40\,keV = 40 \times 10^3\,eV = 6.41 \times 10^{-15}$ Joules

$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$m = 9.1 \times 10^{-31}\,kg$

$$6.41 \times 10^{-15} = \frac{p^2}{2(9.1^{-31})}$$

$p = 1.08 \times 10^{-22}\ kg\,m/s$

then

$$\lambda = \frac{h}{p} = \frac{6.626^{-34}}{1.08^{-22}}$$

$$= 6.14 \times 10^{-12}\,m$$

So one could observe interference & diffraction effects about objects of size $6.14 \times 10^{-12}\,m$

$KE = 100\,keV = 100 \times 10^3\,eV = 1.6 \times 10^{-14}\,J$

then

$p = 1.71 \times 10^{-22}\ kg\,m/s$

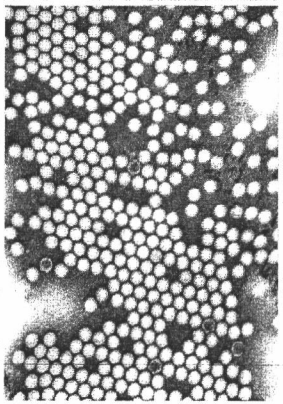
then

$\lambda = 3.88 \times 10^{-12}\,m$

Transmission electron microscopy

From Wikipedia, the free encyclopedia
(Redirected from Transmission electron microscope)

Transmission electron microscopy (TEM) is a microscopy technique whereby a beam of electrons is transmitted through an ultra thin specimen, interacting with the specimen as it passes through. An image is formed from the interaction of the electrons transmitted through the specimen; the image is magnified and focused onto an imaging device, such as a fluorescent screen, on a layer of photographic film, or to be detected by a sensor such as a CCD camera.



A TEM image of the polio virus. The polio virus is 30 nm in size.[1]

TEMs are capable of imaging at a significantly higher resolution than light microscopes, owing to the small de Broglie wavelength of electrons. This enables the instrument's user to examine fine detail—even as small as a single column of atoms, which is tens of thousands times smaller than the smallest resolvable object in a light microscope. TEM forms a major analysis method in a range of scientific fields, in both physical and biological sciences. TEMs find application in cancer research, virology, materials science as well as pollution, nanotechnology, and semiconductor research.

At smaller magnifications TEM image contrast is due to absorption of electrons in the material, due to the thickness and composition of the material. At higher magnifications complex wave interactions modulate the intensity of the image, requiring expert analysis of observed images. Alternate modes of use allow for the TEM to observe modulations in chemical identity, crystal orientation, electronic structure and sample induced electron phase shift as well as the regular absorption based imaging.

The first TEM was built by Max Knoll and Ernst Ruska in 1931, with this group developing the first TEM with resolving power greater than that of light in 1933 and the first commercial TEM in 1939.

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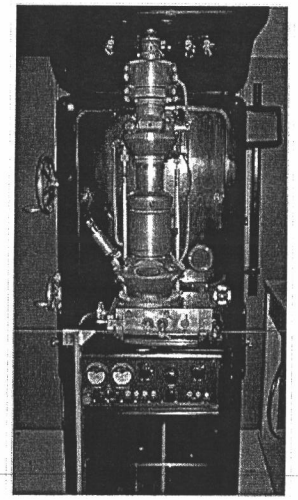
History

Initial development

Ernst Abbe originally proposed that the ability to resolve detail in an object was limited by the wavelength of the light used in imaging, thus limiting the useful obtainable magnification from an optical microscope to a few micrometers. Developments into ultraviolet (UV) microscopes, led by Köhler and Rohr, allowed for an increase in resolving power of about a factor of two.[2] However this required more expensive quartz optical components, due to the absorption of UV by glass. At this point it was believed that obtaining an image with sub-micrometer information was simply impossible due to this wavelength constraint.[3]

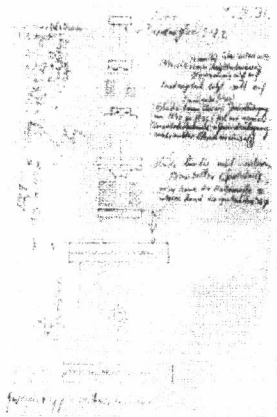
It had earlier been recognized by Plücker in 1858 that the deflection of "cathode rays" (electrons) was possible by the use of magnetic fields.[4] This effect had been utilised to build primitive cathode ray oscilloscopes (CROs) as early as 1897 by Ferdinand Braun, intended as a measurement device.[5] Indeed in 1891 it was recognized by Riecke that the cathode rays could be focused by these magnetic fields, allowing for simple lens designs. Later this theory was extended by Hans Busch in his work published in 1926, who showed that the lens maker's equation, could under appropriate assumptions, be applicable to



The first practical TEM, Originally installed at I. G Farben-Werke and now on display at the Deutsches Museum in Munich, Germany

electrons.[6]

In 1928, at the Technological University of Berlin Adolf Matthias, Professor of High voltage Technology and Electrical Installations, appointed Max Knoll to lead a team of researchers to advance the CRO design. The team consisted of several PhD students including Ernst Ruska and Bodo von Borries. This team of researchers concerned themselves with lens design and CRO column placement, which they attempted to obtain the parameters that could be optimised to allow for construction of better CROs, as well as the development of electron optical components which could be used to generate low magnification (nearly 1:1) images. In 1931 the group successfully generated magnified images of mesh grids placed over the anode aperture. The device used two magnetic lenses to achieve higher magnifications, arguably the first electron microscope. In that same year, Reinhold Rudenberg, the scientific director of the Siemens company, had patented an electrostatic lens electron microscope.[3][7]



Sketch of first electron microscope, originally from Ruska's notebook in 1931, capable of only 16 times magnification

Improving resolution

At this time the wave nature of electrons, which were considered charged matter particles, had not been fully realised until the publication of the De Broglie hypothesis in 1927.[8] The group was unaware of this publication until 1932, where it was quickly realized that the De Broglie wavelength of electrons was many orders of magnitude smaller than that for light, theoretically allowing for imaging at atomic scales. In April 1932, Ruska suggested the construction of a new electron microscope for direct imaging of specimens inserted into the microscope, rather than simple mesh grids or images of apertures. With this device successful diffraction and normal imaging of aluminium sheet was achieved, however exceeding the magnification achievable with light microscopy had still not been successfully demonstrated. This goal was achieved in September 1933, using images of cotton fibers, which were quickly acquired before being damaged by the electron beam.[3]

At this time, interest in the electron microscope had increased, with other groups, such as Albert Prebus and James Hillier at the University of Toronto who constructed the first TEM in North America in 1938,[9] continually advancing TEM design.

Research continued on the electron microscope at Siemens in 1936, the aim of the research was the development improvement of TEM imaging properties, particularly with regard to biological specimens. At this time electron microscopes were being fabricated for specific groups, such as the "EM1" device used at the UK National Physical Laboratory.[10] In 1939 the first commercial electron microscope, pictured, was installed in the Physics department of I. G Farben-Werke. Further work on the electron microscope was hampered by the destruction of a new laboratory constructed at Siemens by an air-raid, as well as the death of two of the researchers, Heinz Müller and Friedrick Krause during World War II.[11]

Further research

After World War II, Ruska resumed work at Siemens, where he continued to develop the electron microscope, producing the first microscope with 100k magnification.[11] The fundamental structure of this microscope design, with multi-stage beam preparation optics, is still used in modern microscopes. The worldwide electron microscopy community advanced with electron microscopes being manufactured in Manchester UK, the USA (RCA), Germany (Siemens) and Japan . The first international conference in electron microscopy was in Delft in 1942, with more than one hundred attendees.[10] Later conferences included the "First" international conference in Paris, 1950 and then in London in 1954.

With the development of TEM, the associated technique of scanning transmission electron microscopy (STEM) was re-investigated and did not become developed until the 1970s, with Albert Crewe at the University of Chicago developing the field emission gun[12] and adding a high quality objective lens to create the modern STEM. Using this design, Crewe demonstrated the ability to image atoms using annular dark-field imaging. Crewe and coworkers at the University of Chicago developed the cold field electron emission source and built a STEM able to visualize single heavy atoms on thin carbon substrates.[13]

Background

Electrons

Theoretically, the maximum resolution, d, that one can obtain with a light microscope has been limited by the wavelength of the photons that are being used to probe the sample, λ and the numerical aperture of the system, NA.[14]

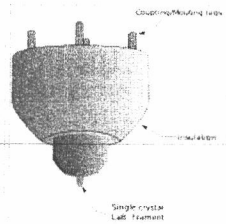
$$d = \frac{\lambda}{2n\sin\alpha} \approx \frac{\lambda}{2\,\mathrm{NA}}$$

Early twentieth century scientists theorised ways of getting around the limitations of the relatively large wavelength of visible light (wavelengths of 400–700 nanometers) by using electrons. Like all matter, electrons have both wave and particle properties (as theorized by Louis-Victor de Broglie), and their wave-like properties mean that a beam of electrons can be made to behave like a beam of electromagnetic radiation. The wavelength of electrons is found by equating the de Broglie equation to the kinetic energy of an electron. An additional correction must be made to account for relativistic effects, as in a TEM an electron's velocity approaches the speed of light, c.[15]

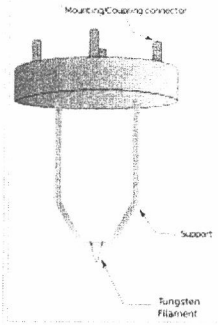
$$\lambda_e \approx \frac{h}{\sqrt{2m_0E\left(1+\frac{E}{2m_0c^2}\right)}}$$

where, h is Planck's constant, m_0 is the rest mass of an electron and E is the energy of the accelerated electron. Electrons are usually generated in an electron microscope by a process known as thermionic emission from a filament, usually tungsten, in the same manner as a light bulb, or alternatively by field electron emission.[16] The electrons are then accelerated by an electric potential (measured in volts) and focused by electrostatic and electromagnetic lenses onto the sample. The transmitted beam contains information about electron density, phase and periodicity; this beam is used to form an image.

Source formation

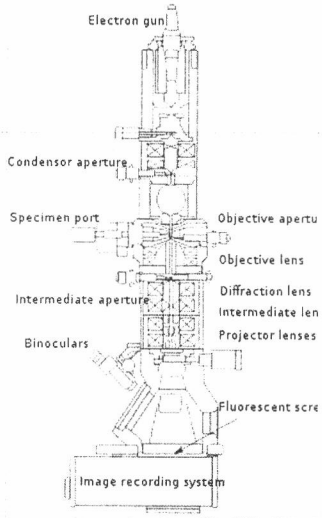


Single crystal LaB_6 filament



Hairpin style tungsten filament

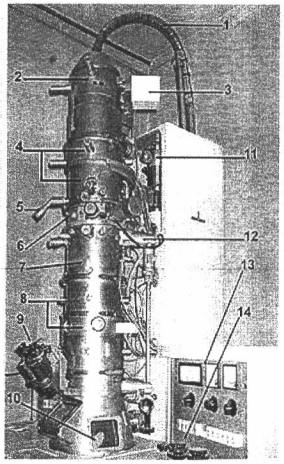
From the top down, the TEM consists of an emission source, which may be a tungsten filament, or a lanthanum hexaboride (LaB_6) source.[17] For tungsten, this will be of the form of either a hairpin-style filament, or a small spike-shaped filament. LaB_6 sources utilize small single crystals. By connecting this gun to a high voltage source (typically ~100-300 kV) the gun will, given sufficient current, begin to emit electrons either by thermionic or field electron emission into the vacuum. This extraction is usually aided by the use of a Wehnelt cylinder. Once extracted, the upper lenses of the TEM allow for the formation of the electron probe to the desired size and location for later interaction with the sample.[18]



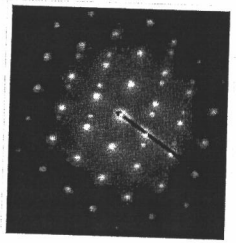
Layout of optical components in a basic TEM

Manipulation of the electron beam is performed using two physical effects. The interaction of electrons with a magnetic field will cause electrons to move according to the right hand rule, thus allowing for electromagnets to manipulate the electron beam. The use of magnetic fields allows for the formation of a magnetic lens of variable focusing power, the lens shape originating due to the distribution of magnetic flux. Additionally, electrostatic fields can cause the electrons to be deflected through a constant angle. Coupling of two deflections in opposing directions with a small

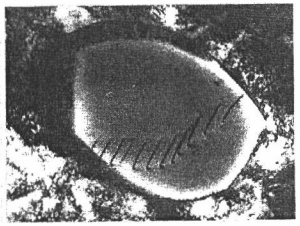
intermediate gap allows for the formation of a shift in the beam path, this being used in TEM for beam shifting, subsequently this is extremely important to STEM. From these two effects, as well as the use of an electron imaging system, sufficient control over the beam path is possible for TEM operation[*citation needed*]. The optical configuration of a TEM can be rapidly changed, unlike that for an optical microscope, as lenses in the beam path can be enabled, have their strength changed, or be disabled entirely simply via rapid electrical switching, the speed of which is limited by effects such as the magnetic hysteresis of the lenses.



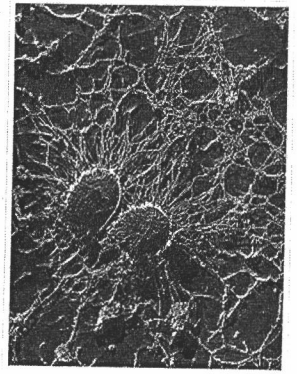
The electron source of the TEM is at the top, where the lensing system (4,7 and 8) focuses the beam on the specimen and then projects it onto the viewing screen (10). The beam control is on the right (13 and 14)



Crystalline diffraction pattern from a twinned grain of FCC Austenitic steel



Transmission electron micrograph of dislocations in steel, which are faults in the structure of the crystal lattice at the atomic scale



Staphylococcus aureus platinum replica image shot on a TEM at 50,000X magnification

5-12 Stanford Linear Accelerator

electrons $KE = 50$ GeV

these are clearly relativistic since $KE \gg mc^2 = 0.511$ MeV

The total energy of the electrons is

$$E_{TOT} = KE + mc^2$$
$$= 50 \text{ GeV} + 0.511 \text{ MeV}$$
$$\doteq 50{,}000 \text{ MeV} + 0.511 \text{ MeV}$$
$$\approx 50{,}000 \text{ MeV}$$

Then use the following expression to obtain momentum

$$E^2 - p^2c^2 = m^2c^4$$
$$(50{,}000)^2 - p^2c^2 = (0.511)^2$$
$$p = 50{,}000 \text{ MeV}/c$$

or

$$pc = 50{,}000 \text{ MeV}$$
$$= 50{,}000 \times 10^6 \text{ eV}$$

Then

$$p = \frac{h}{\lambda}$$
$$pc = \frac{hc}{\lambda}$$

$$hc = 1239.8 \text{ eV} \cdot \text{nm}$$

$$50{,}000 \times 10^6 \text{ eV} = \frac{1239.8 \text{ eV nm}}{\lambda}$$

$$\lambda = 2.48 \times 10^{-8} \text{ nm}$$
$$= 2.48 \times 10^{-17} \text{ m}$$

which is $\frac{1}{100}$th size of proton. This machine studies quarks.

5-18 Tevatron @ FermiLab

$KE = 1$ TeV $= 1000$ GeV $= 1000 \times 10^3$ MeV

proton $mc^2 = 938$ MeV

then as in previous column

$$E_{TOT} = KE + mc^2 = 1000 \times 10^3 \text{ MeV} + 938 \text{ MeV}$$
$$= 1001 \times 10^3 \text{ MeV}$$

then

$$E^2 - p^2c^2 = m^2c^4$$
$$(1001 \times 10^3)^2 - p^2c^2 = (938)^2$$
$$pc = 1.001 \times 10^6 \text{ MeV}$$

then

$$p = \frac{h}{\lambda}$$
$$pc = \frac{hc}{\lambda}$$

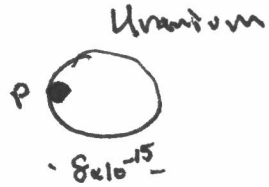
$$hc = 1239.8 \text{ eV nm}$$

$$1.001 \times 10^6 \times 10^6 \text{ eV} = \frac{1239.8 \text{ eV} \cdot \text{nm}}{\lambda}$$

$$\lambda = 1.24 \times 10^{-9} \text{ nm}$$
$$= 1.24 \times 10^{-18} \text{ m}$$

www.ccsem.infn.it/issp2011/docs/Denisov.pdf

5-36



proton
$m = 1.67\times10^{-27}$ kg

Apply uncertainty principle

$$\Delta p\,\Delta x \geq \hbar/2$$

$$\Delta p\,(8\times10^{-15}\,m) \geq \frac{6.626\text{-}34/2\pi}{2}$$

$$\Delta p \geq 6.59\times10^{-21}\ kg\,m/s$$

The book wants us to ASSUME we can use this value as the minimum momentum of the particle. That may or may not be a good assumption

$$P_{min} \doteq 6.59\times10^{-21}\ kg\,m/s^2$$

then if non-relativistic:

$$KE = \tfrac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$= \frac{(6.59\times10^{-21})^2}{2\,(1.67\times10^{-27})}$$

$$= 1.30\times10^{-14}\ \text{Joules}$$

$$= 8.13\times10^{4}\ eV$$

5-42 bacterium

$m = 3\times10^{-15}$ kg
$\Delta x = 1\times10^{-6}$ m

$$\Delta p\,\Delta x \geq \frac{\hbar}{2}$$

$$\Delta p\,(1\times10^{-6}) \geq \frac{6.626\times10^{-34}/2\pi}{2}$$

$$\Delta p = 5.28\times10^{-29}$$

If $p = mv$

$$\Delta p = m\,\Delta v$$

$$5.29\times10^{-29} = 3\times10^{-15}\,\Delta v$$

$$\Delta v = 1.76\times10^{-14}\ m/s$$

which is pretty small

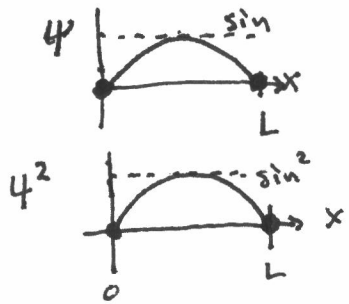
5-48 $\psi = A \sin\left(\frac{\pi x}{L}\right)$

is normalized in a box of width L

Since wavefunctions must be normalized

$$1 = \int_0^L \psi^* \psi \, dx$$

$$1 = \int_0^L A^2 \sin^2\left(\frac{\pi x}{L}\right) dx$$



Use trick since $\sin^2 + \cos^2 = 1$
an sin & cos have same "shape", the integral is 1/2 the area of a box

$$1 = A^2 \underbrace{\tfrac{1}{2}[1 \cdot L]}_{\text{area}}$$

So

$$A = \sqrt{\frac{2}{L}}$$