6-1 Normalize $A e^{i(kx-\omega t)}$

$$1 = \int_{-\infty}^{\infty} \psi^* \psi \, dx$$

$$1 = \int_{-\infty}^{\infty} A e^{-i(kx-\omega t)} A e^{i(kx-\omega t)} dx$$

$$1 = \int_{-\infty}^{\infty} A^2 \, dx$$

This integral makes no sense to evaluate over the range $\pm\infty$. We would get $A = \frac{1}{\sqrt{\infty}}$ for the normalization (aka amplitude)

6-3 Show $\psi = A e^{i(kx-\omega t)}$ satisfies time-dependent Schrö Eqn.

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi = i\hbar \frac{\partial \psi}{\partial t}$$

first: $\frac{\partial \psi}{\partial t} = -i\omega A e^{i(kx-\omega t)}$

then

$$\frac{\partial \psi}{\partial x} = ik A e^{i(kx-\omega t)}$$

$$\frac{\partial^2 \psi}{\partial x^2} = (ik)^2 A e^{i(kx-\omega t)}$$

Then substitute

$$+ + \frac{\hbar^2 k^2}{2m} \cancel{A e^{i(kx-\omega t)}} + V \cancel{A e^{i(kx-\omega t)}} = \hbar\omega \cancel{A e^{i(kx-\omega t)}}$$

$$\frac{\hbar^2 k^2}{2m} + V = \hbar\omega$$

$$\frac{\hbar^2}{2m} + V = E_{TOT}$$

$$KE + V = E_{TOT}$$

6-4 Normalize $A e^{i(kx-\omega t)}$ in the region $x=0$ to a

$$1 = \int_0^a \psi^* \psi \, dx$$

$$1 = \int_0^a A e^{-i(kx-\omega t)} A e^{i(kx-\omega t)} dx$$

$$1 = \int_0^a A^2 dx$$

$$1 = A^2 x \Big|_0^a$$

$$1 = A^2 a$$

so $A = \frac{1}{\sqrt{a}}$

6-5 Normalize $\psi = Are^{-r/\alpha}$ from $r=0$ to ∞

$$1 = \int_0^\infty \psi^* \psi \, dr$$

$$1 = \int_0^\infty Are^{-r/\alpha} \; Are^{-r/\alpha} \, dr$$

$$1 = \int_0^\infty A^2 r^2 e^{-2r/\alpha} \, dr$$

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$$\int_0^\infty x^2 e^{-x/a} = 2a^3$$

in our problem $a = \alpha/2$

$$1 = A^2 \; 2\left(\frac{\alpha}{2}\right)^3$$

$$1 = A^2 \; \cancel{2} \; \frac{\alpha^3}{\cancel{2}\cdot 2\cdot 2}$$

$$A = 2\alpha^{-3/2}$$

Hold it, this is a little strange. If r means radius, then the integral is

$$1 = \int_0^\infty \psi^* \psi \; r^2 dr$$

6-20 Infinite Potential Well.

The allowed Energies are $E_n = n^2 \frac{\pi^2 \hbar^2}{2mL^2}$

a) proton in region $L = 7\times10^{-15}$ m

$m = 1.67\times10^{-27}$ kg

$$E_1 = 1^2 \frac{\pi^2 \left(\frac{6.626-34}{2\pi}\right)^2}{2(1.67-27)(7-15)^2} = 6.71\times10^{-13} \text{ Joules}$$

$$= 4.18\times10^6 \text{ eV}$$

$$= 4.18 \text{ MeV}$$

b) alpha in region $L = 7\times10^{-15}$ m

$m = 4(1.67\times10^{-27}$ kg$)$

Notice that since

$$E_1 = n^2 \frac{\pi^2 \hbar^2}{2mL^2}$$

the alpha will have 1/4 the energy above

$$E_1 = 1.05 \text{ MeV}$$

6-21 Infinite Potential Well

The allowed energies are $E_n = n^2 \frac{\pi^2 \hbar^2}{2mL^2}$

$m = 9.1-31$ kg

$L = 0.5$ nm $= 5-10$ m

$$E_n = n^2\ 2.41\times10^{-19} \text{ J} = n^2\ 1.5 \text{ eV}$$

$E_4 = 24.0$ eV

$E_3 = 13.5$ eV

$E_2 = 6$ eV

$E_1 = 1.5$ eV

The first set of possible jumps are:

Jump $n=4$ to $n=3$

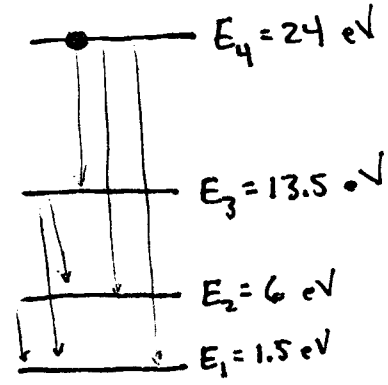
$\Delta E = 24 - 13.5 = 10.5$ eV

Jump $n=4$ to $n=2$

$\Delta E = 24 - 6 = 18$ eV

Jump $n=4$ to $n=1$

$\Delta E = 24 - 1.5 = 22.5$ eV



2nd jump:

Jump $n=3$ to $n=2$

$\Delta E = 13.5 - 6 = 7.5$ eV

Jump $n=3$ to $n=1$

$\Delta E = 13.5 - 1.5 = 12$ eV

Jump $n=2$ to $n=1$

$\Delta E = 6 - 1.5 = 4.5$ eV