7-5 Show R_{20} satisfies radial schrödinger eqn
$n=2 \quad l=0$

$$(7.10) \quad \frac{1}{r^2}\frac{d}{dr} r^2 \frac{dR}{dr} + \frac{2\mu}{\hbar^2}\left\{E - V - \frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2}\right\} R = 0$$

with $l=0$ and $V = \frac{1}{4\pi\epsilon_0}\frac{-e^2}{r}$

$$(7.13) \quad \frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} + \frac{2\mu}{\hbar^2}\left\{E + \frac{e^2}{4\pi\epsilon_0 r}\right\} R = 0$$

$$R_{20} = \left(\frac{1}{2a_0}\right)^{3/2}\left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$$

the constant in front will appear in all three terms when we substitute, so we will drop it.

$$\frac{dR}{dr} = \left(-\frac{1}{a_0}\right) e^{-r/2a_0} + \left(2 - \frac{r}{a_0}\right)\left(-\frac{1}{2a_0}\right) e^{-r/2a_0}$$

$$= \left[-\frac{1}{a_0} - 2\frac{1}{2a_0} + \frac{r}{2a_0^2}\right] e^{-r/2a_0}$$

$$= \left[-\frac{2}{a_0} + \frac{r}{2a_0^2}\right] e^{-r/2a_0}$$

$$= \frac{1}{a_0}\left[-2 + \frac{r}{2a_0}\right] e^{-r/2a_0}$$

$$\frac{d^2R}{dr^2} = \frac{1}{a_0}\left[\frac{1}{2a_0}\right] e^{-r/2a_0} + \frac{1}{a_0}\left[-2 + \frac{r}{2a_0}\right]\left(\frac{-1}{2a_0}\right) e^{-r/2a_0}$$

$$= \frac{1}{a_0}\left(\frac{1}{2a_0}\right)\left[1 + +2 - \frac{r}{2a_0}\right] e^{-r/2a_0}$$

$$= \frac{1}{a_0}\frac{1}{2a_0}\left[3 - \frac{r}{2a_0}\right] e^{-r/2a_0}$$

Substitute into (7.13)
Notice that the exponential will cancel out of each term.

$$\frac{1}{a_0}\frac{1}{2a_0}\left[3 - \frac{r}{2a_0}\right] + \frac{2}{r}\frac{1}{a_0}\left[-2 + \frac{r}{2a_0}\right] + \frac{2\mu}{\hbar^2}\left\{E + \frac{e^2}{4\pi\epsilon_0 r}\right\}\left(2 - \frac{r}{a_0}\right) = 0$$

Multiply by $4a_0^3 r$ to wipe out most of the denominator stuff

$$2\cancel{4}a_0^{\cancel{3}}r \frac{1}{\cancel{a_0}}\frac{1}{\cancel{2a_0}}\left[3 - \frac{r}{2a_0}\right] + 4a_0^{\cancel{3}\,2}r \frac{2}{r}\frac{1}{\cancel{a_0}}\left[-2 + \frac{r}{2a_0}\right] + \frac{2\mu}{\hbar^2}\left\{E + \frac{e^2}{4\pi\epsilon_0 r}\right\} 4a_0^3 r\left(2 - \frac{r}{a_0}\right) = 0$$

$$2a_0 r\left[3 - \frac{r}{2a_0}\right] + 4a_0^2\, 2\left[-2 + \frac{r}{2a_0}\right] + \frac{2\mu}{\hbar^2}\left\{E + \frac{e^2}{4\pi\epsilon_0 r}\right\} 4a_0^2 r\left(2 - \frac{r}{a_0}\right) = 0$$

$$6a_0 r - r^2 + -16a_0^2 + 4a_0 r + \frac{2\mu}{\hbar^2}\left\{E + \frac{e^2}{4\pi\epsilon_0 r}\right\}\left(8a_0^3 r - 4a_0^2 r^2\right) = 0$$

hum...

7-5 CONT

$$-16a_0^2 + 10a_0 r - r^2 \quad + \frac{2\mu}{\hbar^2}\left\{E + \frac{e^2}{4\pi\epsilon_0 r}\right\} 4a_0^2 r\,(2a_0 - r) = 0$$

Maybe the next thing to do is substitute for the total Energy E – book eqn btw 7.14-15

$$E = -\frac{\hbar^2}{2\mu a_0^2}\frac{1}{n^2} \xrightarrow[n=2]{} -\frac{\hbar^2}{8\mu a_0^2}$$

– maybe not this looks like a worse mess.

4-4
2 8

$$-(2a_0 - r)(8a_0 - r)$$

$$-(16a_0^2 - 10a_0 r + r^2)$$

Factor the terms on the left.

$$-\cancel{(2a_0 - r)}(8a_0 - r) + \frac{2\mu}{\hbar^2}\left\{E + \frac{e^2}{4\pi\epsilon_0 r}\right\} 4a_0^2 r\,\cancel{(2a_0 - r)} = 0$$

$$-(8a_0 - r) + \frac{2\mu}{\hbar^2}\left\{E + \frac{e^2}{4\pi\epsilon_0 r}\right\} 4a_0^2 r = 0$$

Multiply this out and collect the powers of r to see what happens

$$-8a_0 + r + \frac{2\mu}{\hbar^2} E\, 4a_0^2 r + \frac{2\mu}{\hbar^2}\frac{e^2}{\cancel{4}\pi\epsilon_0 \cancel{r}}\cancel{4}a_0^2 \cancel{r} = 0$$

$$-8a_0 + \frac{2\mu}{\hbar^2}\frac{e^2}{\pi\epsilon_0} a_0^2 + r + \frac{2\mu}{\hbar^2} E\, 4a_0^2 r = 0$$

$$\left[-8a_0 + \frac{2\mu}{\hbar^2}\frac{e^2}{\pi\epsilon_0} a_0^2\right] + \left[1 + \frac{2\mu}{\hbar^2} E\, 4a_0^2\right] r = 0$$

The only way this can be true is if the stuff in [ ] = 0

$$-8a_0 + \frac{2\mu}{\hbar^2}\frac{e^2}{\pi\epsilon_0} a_0^2 \overset{?}{=} 0$$

cancel a_0

$$-8 + \frac{2\mu}{\hbar^2}\frac{e^2}{\pi\epsilon_0} a_0 \overset{?}{=} 0$$

$$\frac{2\mu}{\hbar^2}\frac{e^2}{\pi\epsilon_0} a_0 \overset{?}{=} 8$$

$$a_0 \overset{?}{=} \frac{8\,\pi\epsilon_0\,\hbar^2}{2\mu e^2}$$

$$a_0 \overset{?}{=} \frac{4\pi\epsilon_0\hbar^2}{\mu e^2}$$

which is also majically true

$$1 + \frac{2\mu}{\hbar^2} E\, 4a_0^2 \overset{?}{=} 0$$

$$E \overset{?}{=} -\frac{\hbar^2}{8\mu a_0^2}$$

which is majically true

OK, that was somewhat painful.

7-6 Show R_{21} satisfies 7.10

$n=2 \quad \ell=1$

$$(7.10) \qquad \frac{1}{r^2}\frac{d}{dr} r^2 \frac{dR}{dr} + \frac{2\mu}{\hbar^2}\left\{E - V - \frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2}\right\} R = 0$$

with $\ell = 1$

$$\#1 \qquad \frac{1}{r^2}\frac{d}{dr} r^2 \frac{dR}{dr} + \frac{2\mu}{\hbar^2}\left\{E - V - \frac{\hbar^2}{2\mu}\frac{2}{r^2}\right\} R = 0$$

$$R_{21} = \underbrace{\frac{1}{\sqrt{3}\,(2a_0)^{3/2}}\frac{1}{a_0}}_{\text{overall constant factor}} r e^{-r/2a_0}$$

$\therefore$ drop because it will cancel out of every term.

use $R_{21} \sim r e^{-r/2a_0}$

$$\frac{dR}{dr} \sim e^{-r/2a_0} + r\left(-\frac{1}{2a_0}\right)e^{-r/2a_0}$$

$$\sim \left(1 - \frac{r}{2a_0}\right) e^{-r/2a_0}$$

then

$$r^2 \frac{dR}{dr} \sim \left(r^2 - \frac{r^3}{2a_0}\right) e^{-r/2a_0}$$

and then

$$\frac{d}{dr} r^2 \frac{dR}{dr} \sim \left(2r - \frac{3r^2}{2a_0}\right)e^{-\frac{r}{2a_0}} + \left(r^2 - \frac{r^3}{2a_0}\right)\left(-\frac{1}{2a_0}\right)e^{-\frac{r}{2a_0}}$$

$$\sim \left[2r - \frac{3}{2a_0}r^2 + \left(-\frac{1}{2a_0}\right)r^2 + \left(\frac{1}{2a_0}\right)^2 r^3\right] e^{-r/2a_0}$$

multiply #1 by r^2

$$\frac{d}{dr} r^2 \frac{dR}{dr} + \frac{2\mu}{\hbar^2}\left\{E - V - \frac{\hbar^2}{2\mu}\frac{2}{r^2}\right\} r^2 R = 0$$

Substitute

$$\left[2r - \frac{3}{2a_0}r^2 + \left(\frac{-1}{2a_0}\right)r^2 + \left(\frac{1}{2a_0}\right)^2 r^3\right] e^{-r/2a_0} + \frac{2\mu}{\hbar^2}\left\{E - V - \frac{\hbar^2}{2\mu}\frac{2}{r^2}\right\} r^2 r e^{-r/2a_0} = 0$$

$$\left[\left(\frac{1}{2a_0}\right)^2 r^3 - \left(\frac{3}{2a_0} + \frac{1}{2a_0}\right)r^2 + 2r \qquad + \frac{2\mu}{\hbar^2}\left\{E - V - \frac{\hbar^2}{2\mu}\frac{2}{r^2}\right\} r^3 = 0\right.$$

$$-\left(\frac{4}{2a_0}\right)$$

$$-\left(\frac{2}{a_0}\right)$$

Substitute $V = \frac{-e^2}{4\pi\epsilon_0 r}$, multiply out and collect powers of r $\square$

7.6 CONT

$$\left(\frac{1}{2a_0}\right)^2 r^3 + \left(-\frac{2}{a_0}\right) r^2 + 2r + \frac{2\mu}{\hbar^2}\left\{E + \frac{e^2}{4\pi\epsilon_0 r} - \frac{\hbar^2}{2\mu}\frac{2}{r^2}\right\} r^3 = 0$$

$$\left(\frac{1}{2a_0}\right)^2 r^3 + \left(\frac{-2}{a_0}\right) r^2 + 2r + \frac{2\mu}{\hbar^2} E r^3 + \frac{2\mu}{\hbar^2}\frac{e^2}{4\pi\epsilon_0} r^2 - \frac{2}{r^2} r^3 = 0$$

The r^3 terms must go to zero

$$\left(\frac{1}{2a_0}\right)^2 + \frac{2\mu}{\hbar^2} E = 0$$

$$E = -\frac{\hbar^2}{2\mu}\left(\frac{1}{2a_0}\right)^2$$

$$= \frac{-\hbar^2}{8\mu a_0^2}$$

which is what we expect the energy to be.

The r^2 terms must go to zero

$$\frac{-2}{a_0} + \frac{2\mu}{\hbar^2}\frac{e^2}{4\pi\epsilon_0} \stackrel{?}{=} 0$$

$$\frac{2\mu_0}{\hbar^2}\frac{e^2}{4\pi\epsilon_0} \stackrel{?}{=} \frac{2}{a_0}$$

$$\frac{\hbar^2 4\pi\epsilon_0}{2\mu_0 e^2} \stackrel{?}{=} \frac{1}{2} a_0$$

$$\frac{4\pi\epsilon_0 \hbar^2}{\mu_0 e^2} \stackrel{?}{=} a_0$$

which is, what we used for the definition of a_0

7-7 Show that R_{21} is normalized

$$R_{21} = \frac{1}{\sqrt{3}(2a_0)^{3/2}} \frac{1}{a_0} r e^{-r/2a_0}$$

The normalization condition for a 3-Dim Radial Wfn is

$$\int_0^\infty R^* R \, r^2 \, dr = 1$$

subst

$$\frac{1}{3(2a_0)^3} \left(\frac{1}{a_0}\right)^2 \int_0^\infty r^2 e^{-r/a_0} \, r^2 \, dr$$

$$\frac{1}{3(2a_0)^3} \left(\frac{1}{a_0}\right)^2 \int_0^\infty r^4 e^{-r/a_0} \, dr$$

See Appendix 3 page A-4

$$\int_0^\infty x^n e^{-x/\alpha} \, dx = n! \, \alpha^{n+1}$$

$$\frac{1}{3(2a_0)^3} \left(\frac{1}{a_0}\right)^2 \quad 4! \, a_0^{4+1}$$

$$\frac{1}{3(2a_0)^3} \left(\frac{1}{a_0}\right)^3 \quad 4! \, a_0^5$$

$$\cancel{\frac{1}{3 \cdot 2^3 \, a_0^8}} \quad 4 \cdot \cancel{3} \cdot 2 \cdot 1 \, \cancel{a_0^5}$$

$$\underline{1}$$

yeah!