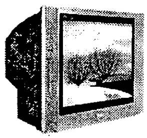
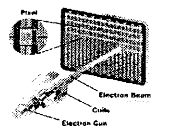
Example 2.11

mc2 = 0.511 MeV
m = 9.1e-31 kg
|q| = 1.6e-19 Coul

Electrons in a television set are accelerated by a potential difference of 25000 Volts before striking the screen.

a). Calculate the speed of the electrons and
b). Determine the error in using the classical kinetic energy result.



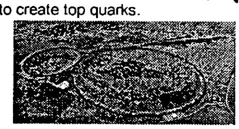


http://express.howstuffworks.com/exp-tv1.htm

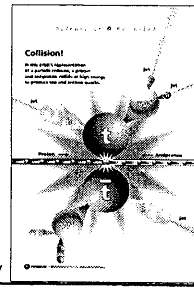
http://www.o-digital.com/wholesale-products/2227/2285-4/LCD-TV-LDT32-225837.html

Example 2.13 Fermilab

A 2-GeV proton hits another 2-GeV proton in a head-on collision in order to create top quarks.



http://www.fnal.gov



- For each of the initial protons, calculate
  - Speed v
  - β
  - Momentum p
  - Rest-mass Energy
  - Kinetic Energy KE
  - Total Energy E_{tot}

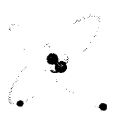
mc^2=938 MeV

Example 2.16

The helium nucleus is built from 2 protons and 2 neutrons. The *binding energy* is the difference in rest mass-energy of the nucleus from the total rest mass-energy of it's component parts.

Calculate the *nuclear binding energy* of helium.

m_{He} = 4.002603 amu
m_p = 1.007825 amu
m_n = 1.008665 amu



Hints:
1 amu = 1.67e-27 kg
or
c^2 = 931.5 MeV/amu

http://www.dbxsoftware.com/helium/

Example 2.17

The molecular binding energy is called dissociation energy. It is the energy required to separate the atoms in a molecule. The dissociation energy of the NaCl molecule is 4.24 eV.

Determine the fractional mass increase of the Na and Cl atoms when they are not bound together in NaCl.

m_{Na} = 22.98976928 amu

Average m_{Cl} = 35.453 amu

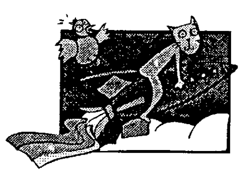
Hints:
1 amu = 1.67e-27 kg
or
c^2 = 931.5 MeV/amu

http://www.ionizers.org/water.html

Sandin 5.30

A spaceship has a length of 100 m and a mass of 4e+9 kg as measured by the crew. When it passes us, we measure the spaceship to be 75 m long.

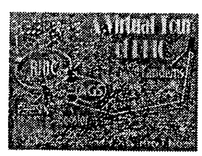
What do we measure its momentum to be?



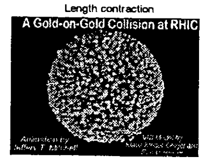
RHIC
relativistic heavy ion collider

The diameter of an gold nucleus is 14 fm.

If a Au nucleus has a kinetic energy of 4000 GeV, what is the apparent 'thickness' of the nucleus in the laboratory?



Length contraction



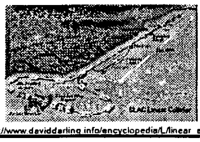
mc^2=197*931.5 MeV

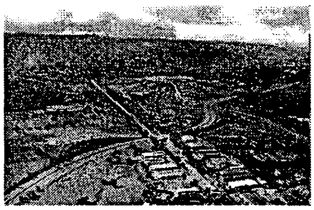
http://www.bnl.gov/rhic/

Sandin 5.22

At the Stanford Linear Accelerator, 50 GeV electrons are produced

- For one of these electrons, calculate
  - Speed v
  - β
  - Momentum p
  - Rest-mass Energy
  - Kinetic Energy KE
  - Total Energy E_{tot}





http://www.flickr.com/photos/kqedquest/3268446670/

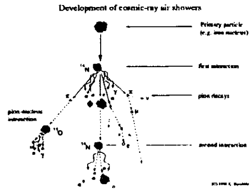
http://www.daviddarling.info/encyclopedia/L/linear_accelerator.html

$mc^2 = 0.511$ MeV

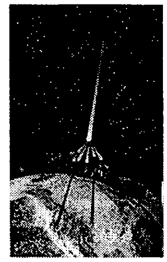
Sandin 5.25

A cosmic ray pion (rest mass 140 MeV/c^2) has a momentum of 100 MeV/c.

http://www.mpi-hd.mpg.de/hfm/CosmicRay/Showers.html



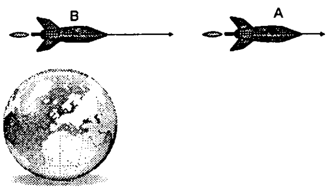
- Calculate
  - Speed v
  - β
  - Momentum p
  - Rest-mass Energy
  - Kinetic Energy KE
  - Total Energy E_{tot}



http://www2.slac.stanford.edu/vvc/cosmicrays/cratmos.html

Sandin 4.26

Spaceship A moves past us at 0.6c
followed by Spaceship B in the same direction at 0.8c

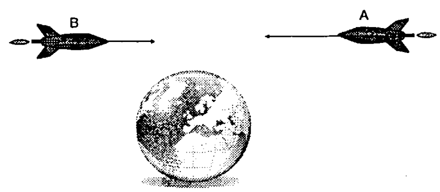


What do *they* measure as their relative speed of approach?

What do *we* measure as their relative speed of approach?

Sandin 4.28

Spaceship A approaches us from the right at at 0.8c
Spaceship B approaches us from the left at 0.6c



What do *they* measure as their relative speed of approach?

What do *we* measure as their relative speed of approach?

Rex Thorton

2.11

a) $KE = qV = 25000\ eV = 25\ keV = 0.025\ MeV$

then

$$KE = (\gamma - 1)\, mc^2$$
$$0.025 = (\gamma - 1)\ 0.511$$
$$\gamma = 1.049$$
$$\frac{1}{\sqrt{1-\beta^2}} = 1.049$$
$$\beta = 0.3018 \quad \text{or} \quad u = 9.055 \times 10^7\ m/s$$

b) $KE = qV = 25000\ eV =$

then

$$KE = \tfrac{1}{2} m v^2$$
$$= \tfrac{1}{2} mc^2 \frac{v^2}{c^2}$$
$$= \tfrac{1}{2} mc^2 \beta^2$$
$$0.025 = \tfrac{1}{2}\ 0.511\ \beta^2$$
$$\beta = 0.3128$$

So the %-difference is $\frac{0.3128 - 0.3018}{0.3018} = 0.036 \approx 4\%$

Rex Thorton

2.13 Fermi Lab $KE = 2\ GeV = 2000\ MeV$

a) & b)

$$KE = (\gamma - 1)\, mc^2$$
$$2000 = (\gamma - 1)\ 938$$
$$\gamma = 3.132$$
$$\frac{1}{\sqrt{1-\beta^2}} = 3.132$$
$$\beta = 0.948 \quad \text{or} \quad u = (0.948)(3\times 10^8) = 2.84 \times 10^8\ m/s$$

c)

$$p = \gamma m u$$
$$= \gamma\, mc^2\ \frac{u}{c}\ \frac{1}{c}$$
$$= \gamma\, mc^2\ \beta\ \frac{1}{c}$$
$$= (3.132)(938\ MeV)\ 0.948\ \frac{1}{c}$$
$$= 2.785 \times 10^3\ MeV/c$$

d) $mc^2 = 938\ MeV$ as given

e) $KE = 2000\ MeV$ as given

f) $E_{TOT} = KE + mc^2 = 2000 + 938 = 2938\ MeV$

Rex Thorton

2.16 Binding Energy

nucleus rest mass energy = $m_{He}c^2 = (4.002603\,\text{amu})(931.5\,\text{MeV/amu})$

$= 3.728425 \times 10^3\ \text{MeV}$

components rest mass energy = $2m_pc^2 + 2m_nc^2$

$= 2(1.007825)(931.5) + 2(1.008665)(931.5)$

$= 3.7567208 \times 10^3\ \text{MeV}$

Binding energy = 28.3 MeV

Rex Thorton

2.17 Dissociation Energy

The energy change of $4.24\ \text{eV} = 4.24 \times 10^{-6}\ \text{MeV}$ corresponds to a mass change of

$$\Delta E_{diss} = \Delta m c^2$$

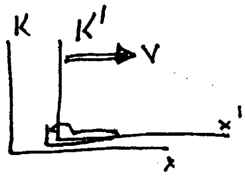
$$(4.24 \times 10^{-6}\ \text{MeV}) = \Delta m\ (931.5\ \text{MeV/amu})$$

$$\Delta m = 4.55 \times 10^{-9}\ \text{amu}$$

The percentage difference is

$$\frac{\Delta m}{m_{Na} + m_{Cl}} = \frac{4.55 \times 10^{-9}}{22.99 + 35.45} = 7.8 \times 10^{-11}$$

Sandin 5.30



length contraction –
– we observe the ship to be shorter

$$L = \frac{L_0}{\gamma}$$

$$75 = \frac{100}{\gamma}$$

$$\gamma = 1.333$$

$$\frac{1}{\sqrt{1-\beta^2}} = 1.333$$

$$\beta = 0.661$$

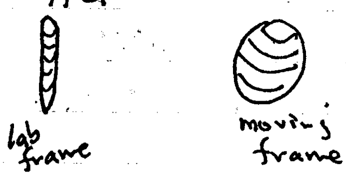
Then

$$p = \gamma m u$$

$$= 1.333\ (4 \times 10^9\ kg)\ [0.661 \times 3 \times 10^8]$$

$$= 1.058 \times 10^{18}\ kg\ m/s$$

RHIC Au



KE = 4000 GeV
= 4000,000 MeV

$$KE = (\gamma - 1)\, mc^2$$

$$4000{,}000 = (\gamma - 1)\, [197 \times 931.5]$$

$$\gamma = 22.8$$

length contraction

$$L = \frac{L_0}{\gamma}$$

$$= \frac{14\ fm}{22.8} = 0.61\ fm$$

Sandin 5.22

$KE = 50\ GeV = 50{,}000\ MeV$
$mc^2 = 0.511\ MeV$

a) Speed = β

$KE = (\gamma - 1)\, mc^2$
$50000 = (\gamma - 1)\, 0.511$
$\gamma = 9.785 \times 10^4$
$\frac{1}{\sqrt{1-\beta^2}} = 9.785 \times 10^4$
$\beta = 0.999999999995 = 1.00$

so $u = 1.00$

b) Momentum

$p = \gamma m u = \gamma\, mc^2 \frac{u}{c^2}$
$= \gamma\, mc^2 \frac{u}{c} \frac{1}{c}$
$= (9.785 \times 10^4)(0.511\ MeV)(1.00) \frac{1}{c}$
$= 5.00 \times 10^4\ MeV/c$
$= 50{,}001\ MeV/c$
$= 50.001\ GeV/c$

c) Rest Mass Energy

given $mc^2 = 0.511\ MeV$

d) $KE = 50{,}000\ MeV$ given

e) Total $E = KE + mc^2 = 50{,}000 + 0.511$
$\simeq 50{,}000\ MeV$

Sandin 5.25 Pion

$p = 100$ MeV/c momentum

$mc^2 = 140$ MeV rest-mass energy

Momentum

$$p = 100 \text{ MeV}/c$$
$$p = \gamma m u$$
$$= \gamma\, mc^2 \frac{u}{c^2}$$
$$= \gamma\, mc^2 \frac{u}{c} \cdot 1/c$$
$$100 \frac{\text{MeV}}{c} = \gamma\,(140 \text{ MeV})\ \beta\ 1/c$$
$$0.714 = \gamma\beta$$
$$= \frac{1}{\sqrt{1-\beta^2}}\ \beta$$
$$(0.714)^2 = \frac{1}{1-\beta^2}\ \beta^2$$
$$\frac{1-\beta^2}{\beta^2} = \left(\frac{1}{0.714}\right)^2$$
$$= 1.96$$
$$\frac{1}{\beta^2} - 1 = 1.96$$
$$\frac{1}{\beta^2} = 2.96$$
$$\beta = 0.58$$

so $V = 0.58\ (3\times10^8)$
$= 1.74\times10^8$ m/s

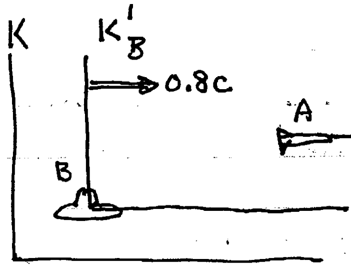
Total Energy

$$E_{TOT}^2 - p^2c^2 = m^2c^4$$
$$E_{TOT}^2 = p^2c^2 + (mc^2)^2$$
$$= \left(100 \frac{\text{meV}}{c}\right)^2 c^2 + (140 \text{ MeV})^2$$
$$E_{Tot} = 172 \text{ meV}$$

Kinetic Energy

$$E_{TOT} = KE + mc^2$$
$$172 = KE + 140$$
$$KE = 32 \text{ MeV}$$

Sandin 4.26



$V = 0.8c$
$= 2.4 \times 10^8$ m/s

$\beta = 0.8$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} = 1.67$

a) We want to jump + ride along on spaceship B

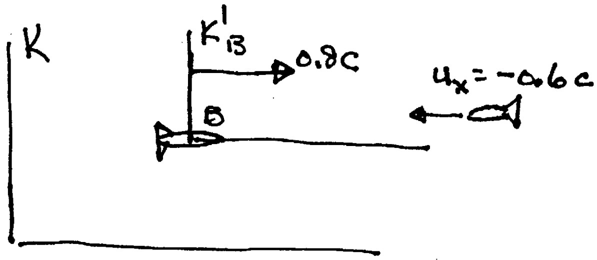
$$u_x' = \frac{u_x - V}{1 - \frac{\beta}{c} u_x}$$

$$= \frac{0.6c - 0.8c}{1 - \frac{0.8}{c} 0.6} = \frac{-0.2c}{0.52}$$

$$= -0.38c$$ which sounds weird

b) We observe the closure rate to be the difference in speeds : 0.2c

Sandin 4.28



$V = 0.8c$

$\beta = 0.8$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} = 1.67$

a) We want to jump + ride with spaceship B

$$u_x' = \frac{u_x - V}{1 - \frac{\beta}{c} u_x}$$

$$= \frac{-0.6c - 0.8c}{1 - \frac{0.8}{c}(-0.6c)} = \frac{-1.4c}{1.48}$$

$$= 0.946\ c$$

b) We observe the closure rate to be the difference in speeds — which is actually 1.4 c !!

This OK because <u>Noting is actually moving</u> at that speed.