MULTIPLE CHOICE (20 POINTS)

Answer **all** of the following questions.

1. Each of two small non-conducting spheres is charged positively, the combined charge being 40 μC. When the two spheres are 50 cm apart, each sphere is repelled by the other by a force of magnitude 2.0 N. Determine the magnitude of the smaller of the two charges.

   (a) 1.4 μC
   (b) 1.1 μC
   (c) 2.0 μC
   (d) 3.3 μC
   (e) 17 μC

   SOLUTION

   The product of the charges must satisfy

$$2 = k\frac{q_1 q_2}{(0.5)^2}. \tag{1}$$

   Since the charges sum to 40 μC, we may make the substitution

$$q_2 = 4 \times 10^{-5} - q_1, \tag{2}$$

   and solve for q_1. This yields the two roots

$$q_1 = \{3.9 \times 10^{-5}, 1.4 \times 10^{-6}\}, \tag{3}$$

   the smaller of which may be taken as the magnitude of the smaller charge.

2. A particle ($m = 50$ g, $q = 5.0$ μC) is released from rest when it is 50 cm from a second particle ($Q = -20$ μC). Determine the magnitude of the initial acceleration of the 50-g particle.

   (a) 54 m/s^2
   (b) 90 m/s^2
   (c) 72 m/s^2
   (d) 65 m/s^2
   (e) 36 m/s^2

   SOLUTION

   The only force acting on the particle is the electric force, thus

$$F = ma = q_1 E. \tag{4}$$

   Solving for a, we have

$$\begin{aligned} a &= \frac{|q_1|E}{m} && (5) \\ &= \frac{k|q_1||q_2|}{r^2 m} && (6) \\ &= \frac{(9 \times 10^9)(5 \times 10^{-6})(2 \times 10^{-5})}{(0.5)^2(0.05)} && (7) \\ &= 72. && (8) \end{aligned}$$

3. A +20-nC point charge is placed on the x-axis at $x = 2.0$ m, and a −25-nC point charge is placed on the y-axis at $y = -3.0$ m. What is the direction of the electric field at the origin?

   (a) 209°
   (b) 61°
   (c) 29°
   (d) 241°
   (e) 151°

   SOLUTION

   By inspection, we conclude that the resulting field vector must lie in the third quadrant, and the angle it makes with the x-axis is

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{E_y}{E_x}\right) &(9)\\ &= \tan^{-1}\left(\frac{\frac{\cancel{k}|q_-|}{y^2}}{\frac{\cancel{k}|q_+|}{x^2}}\right) &(10)\\ &= \tan^{-1}\left(\frac{\frac{2.5\cancel{\times 10^{-8}}}{9}}{\frac{2.0\cancel{\times 10^{-8}}}{4}}\right) &(11)\\ &= \tan^{-1}\left(\frac{5}{9}\right) &(12)\\ &= 29^\circ. &(13) \end{aligned}$$

   For a vector in the third quadrant, this corresponds to an angle of 209° from the positive x-axis.

4. A point charge (5.0 pC) is located at the center of a spherical surface (radius = 2.0 cm), and a charge of 3.0 pC is spread uniformly upon this surface. Determine the magnitude of the electric field 1.0 cm from the point charge.

   (a) 0.72 kN/C
   (b) 0.45 kN/C
   (c) 0.63 kN/C
   (d) 0.90 kN/C
   (e) 0.18 kN/C

   SOLUTION

   The field inside the spherical shell is due only to the point charge at the center, given by

$$E = k_{\mathsf{e}}\frac{|q|}{r^2} \tag{14}$$

$$= (9 \times 10^9)\frac{5 \times 10^{-12}}{(0.01)^2} \tag{15}$$

$$= 450. \tag{16}$$

5. Which one of the following is not an expression for electric charge?

   (a) $\int \rho \, dV$

   (b) $\int \sigma \, dA$

   (c) $\int \lambda \, dl$

   (d) $\int \mathbf{E} \cdot d\mathbf{A}$

   (e) $\epsilon_0 \int \mathbf{E} \cdot d\mathbf{A}$

   SOLUTION

   The expression

   $$\int \mathbf{E} \cdot d\mathbf{A} \tag{17}$$

   has units of N·m^2/C, and so cannot represent electric charge.

6. The electric field just outside the surface of a hollow conducting sphere of radius 20 cm has a magnitude of 500 N/C and is directed outward. A charge Q is introduced into the center of the sphere and it is noted that the electric field is still directed outward but has decreased to 100 N/C. What is the magnitude of the charge Q?

   (a) 1.5 nC
   (b) 1.8 nC
   (c) 1.3 nC
   (d) 1.1 nC
   (e) 2.7 nC

   SOLUTION

   By superposition we have

$$100 = E_{\mathsf{sphere}} + E_{\mathsf{Q}} \tag{18}$$

$$= 500 + k\frac{Q}{(0.2)^2}. \tag{19}$$

   Solving for Q, we have

$$Q = -\frac{400(0.2)^2}{k} = -1.8 \times 10^{-9}. \tag{20}$$

7. An electron ($m = 9.1 \times 10^{-31}$ kg, $q = -1.6 \times 10^{-19}$ C) starts from rest at point A and has a speed of 5.0×10^{6} m/s at point B. Only electric forces act on it during this motion. Determine the electric potential difference $V_{\mathsf{A}} - V_{\mathsf{B}}$.

   (a) -71 V
   (b) $+71$ V
   (c) -26 V
   (d) $+26$ V
   (e) -140 V

   SOLUTION

   The potential difference is given by

$$\Delta V = \frac{\Delta U}{q} \tag{21}$$

$$= \frac{mv^2}{2q} \tag{22}$$

$$= \frac{(9.1 \times 10^{-31})(5.0 \times 10^{6})^2}{2(-1.6 \times 10^{-19})} \tag{23}$$

$$= -71. \tag{24}$$

   To check the sign of our answer, we remember that in an electric field, a negative charge like an electron will travel in the opposite direction of the electric field lines (that is, from lower to higher potential). The quantity $\Delta V = V_{\mathsf{A}} - V_{\mathsf{B}}$ should indeed be negative.

8. A charge of uniform linear density 6.0 nC/m is distributed along the x-axis from $x = 0$ to $x = +3$ m. Which of the following integrals is correct for the electric potential (relative to zero at infinity) at the point $y = +4$ m on the y-axis.

   (a) $\int_0^3 \frac{54dx}{\sqrt{x^2+16}}$

   (b) $\int_0^3 \frac{18dx}{\sqrt{x^2+16}}$

   (c) $\int_0^3 \frac{54dx}{x^2+16}$

   (d) $\int_0^3 \frac{18dx}{x^2+16}$

   (e) $\int_0^3 \frac{108dx}{\sqrt{x^2+16}}$

SOLUTION

The potential due to the charge distribution is given by

$$V = k\int \frac{dq}{r} \tag{25}$$

$$= k\int_{x_1}^{x_2} \frac{\lambda dx}{\sqrt{x^2+y^2}} \tag{26}$$

$$= \int_0^3 \frac{(9\times 10^9)(6\times 10^{-9})dx}{\sqrt{x^2+4^2}} \tag{27}$$

$$= \int_0^3 \frac{54dx}{\sqrt{x^2+16}} \tag{28}$$

9. Three identical point charges (+2.0 nC) are placed at the corners of an equilateral triangle with sides of 2.0-m length. If the electric potential is taken to be zero at infinity, what is the potential at the midpoint of any one of the sides of the triangle?

   (a) 16 V
   (b) 10 V
   (c) 70 V
   (d) 46 V
   (e) 44 V

   SOLUTION

   By superposition, the potential is

$$V = kq\left(\frac{1}{1}+\frac{1}{1}+\frac{1}{\sqrt{3}}\right) \quad (29)$$

$$= (9\times 10^{9})(2\times 10^{-9})(2.58) \quad (30)$$

$$= 46. \quad (31)$$

10. What is the magnitude of the electric field at the point $(3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k})$ m if the electric potential is given by $V = 2xyz^2$, where V is in volts, and x, y, and z are in meters?

    (a) 150 N/C
    (b) 96 N/C
    (c) 64 N/C
    (d) 22.5 kN/C
    (e) There is no field at that point.

SOLUTION

The field at any point is given by

$$\begin{aligned} \mathbf{E} &= -\nabla V & (32) \\ &= -\left(\frac{\partial V}{\partial x}\mathbf{i} + \frac{\partial V}{\partial y}\mathbf{j} + \frac{\partial V}{\partial z}\mathbf{k}\right) & (33) \\ &= -\left[\frac{\partial}{\partial x}(2xyz^2)\mathbf{i} + \frac{\partial}{\partial y}(2xyz^2)\mathbf{j} + \frac{\partial}{\partial z}(2xyz^2)\mathbf{k}\right] & (34) \\ &= -2yz^2\mathbf{i} - 2xz^2\mathbf{j} - 4xyz\mathbf{k}. & (35) \end{aligned}$$

At $(3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k})$, this is

$$\mathbf{E} = 64\mathbf{i} - 96\mathbf{j} + 96\mathbf{k}. \tag{36}$$

The magnitude is given by

$$E = \sqrt{64^2 + 96^2 + 96^2} = 150. \tag{37}$$

OPEN ENDED ANSWER (5 POINTS)

Choose **one** of the following problems. Check the box next to the problem for which you intend to submit a solution.

- ❑ Imagine for a minute that the moon is held in its orbit about the Earth by electrical forces rather than by gravitation. What electrical charges $-Q$ on the Earth and $+Q$ on the moon are necessary to hold the moon in a circular orbit with a period of 27.3 days? The Earth-moon distance is 384 000 km and the mass of the moon is 7.35×10^{22} kg.
- ❑ A Geiger counter is like an electroscope that discharges whenever ions formed by a radioactive particle produce a conducting path. A typical Geiger counter consists of a conducting wire of radius 0.002 cm stretched along the axis of a hollow conducting cylinder of radius 2.0 cm. The wire and the cylinder carry equal and opposite charges of 8×10^{-10} C all along their length of 10.0 cm. What is the magnitude of the electric field at the surface of the wire?
- ❑ The gap between electrodes in a spark plug is 0.06 cm. In order to produce an electric spark in a gasoline-air mixture, the electric field must reach a value of 3×10^{6} V/m. What minimum voltage must be supplied by the ignition circuit when starting the car?

BONUS (2 POINTS)

Derive the formula for the capacitance of two capacitors in series.