Due Wednesday, 16 January, 2008.

1 Calculus Refresher

Evaluate the integrals:

$$\int_0^2 (6x^2 - 4x + 5)dx = \boxed{18} \tag{1}$$

$$\int_{-1}^0 (2x - e^x)dx = \boxed{-2 + \frac{1}{e}} \tag{2}$$

$$\int_{-2}^2 (3u + 1)^2 du = \boxed{52} \tag{3}$$

$$\int_1^4 \sqrt{t}(1 + t)dt = \boxed{\frac{256}{15}} \tag{4}$$

$$\int_{-2}^{-1} \left(4y^3 + \frac{2}{y^3}\right) dy = \boxed{-\frac{63}{4}} \tag{5}$$

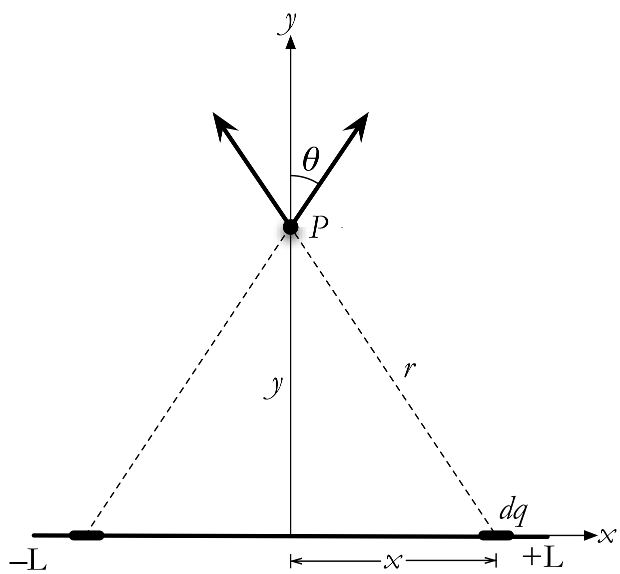
$$\int_1^4 \sqrt{\frac{5}{x}} dx = \boxed{2\sqrt{5}} \tag{6}$$

$$\int_0^\pi (4\sin\theta - 3\cos\theta)d\theta = \boxed{8} \tag{7}$$

$$\int_0^{\pi/4} \frac{1 + \cos^2\theta}{\cos^2\theta} d\theta = \boxed{1 + \frac{1}{\pi}} \tag{8}$$

2 Linear Charge Distribution

Find the electric field a distance y above the midpoint of a straight line segment of length $2L$, which carries a uniform linear charge density λ. (Hint: it is advantageous to chop the line up into symmetrically placed pairs (at $\pm x$), for then the horizontal components of the two fields cancel.)



SOLUTION

Since the horizontal field components cancel, the net field of each charge pair is

$$d\mathbf{E} = 2k\frac{dq}{r^2}\cos\theta\mathbf{j} \tag{9}$$

$$= 2k\frac{\lambda dx}{r^2}\cos\theta\mathbf{j}. \tag{10}$$

We recognize that

$$\cos\theta = \frac{y}{r} = \frac{y}{\sqrt{y^2+x^2}}, \tag{11}$$

and we write

$$\begin{aligned}
\mathbf{E} &= \int_0^L 2k\frac{\lambda y}{(y^2+x^2)^{3/2}}dx\mathbf{j} && (12)\\
&= 2k\lambda y\int_0^L \frac{1}{(y^2+x^2)^{3/2}}dx\mathbf{j} && (13)\\
&= 2k\lambda y\left[\frac{x}{y^2\sqrt{y^2+x^2}}\right]_0^L\mathbf{j} && (14)\\
&= \boxed{k\frac{2\lambda L}{y\sqrt{y^2+L^2}}\mathbf{j}}. && (15)
\end{aligned}$$

For $y \gg L$, this simplifies to

$$\mathbf{E} \approx k\frac{2\lambda L}{y^2}, \tag{16}$$

i.e. the line looks like a point charge $q = 2\lambda L$ a distance y from point P.

3 Serway Problem 23.27

Hint: $dq = \lambda r d\theta$.

SOLUTION

Using the hint, and recognizing that all the y-components cancel, we write

$$\begin{aligned}
\int dE_x &= \frac{kdq}{r^2}\cos\theta && (17)\\
&= \int \frac{k\lambda r\cos\theta}{r^2}d\theta && (18)\\
&= \frac{k\lambda}{r}\int_{-\pi/2}^{\pi/2}\cos\theta d\theta && (19)\\
&= \frac{k\lambda}{r}\sin\theta\Big|_{-\pi/2}^{\pi/2} && (20)\\
&= \frac{2k\lambda}{r}. && (21)
\end{aligned}$$

Since the total charge $Q = \lambda l$ and $r = l/\pi$, we write

$$\mathbf{E} = \frac{2\pi k Q}{l^2}\mathbf{i} \tag{22}$$

$$= \frac{2\pi(9\times10^{9})(-7.5\times10^{-6})}{(0.14)^2}\mathbf{i} \tag{23}$$

$$= \boxed{-2.16\times10^{7}\mathbf{i}}. \tag{24}$$