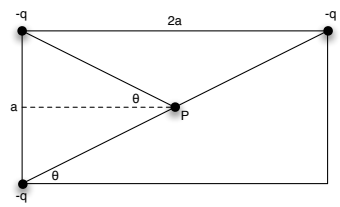
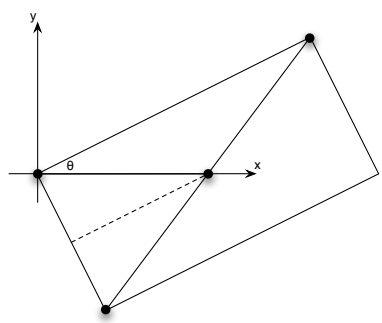
This was an optional homework assignment. As promised, however, one of the problems was presented on the quiz...the Gauss' Law problem!

1 Electric Field

Three charges of magnitude $-q$ are located at three corners of a rectangle whose sides are length a and $2a$. What is the electric field in the center of the rectangle?

SOLUTION





The important thing to recognize in this problem is that the two charges on opposite corners of the rectangle create fields that are equal and opposite (see figure). The problem is then simply to find the field at P due to the single remaining point charge $-q$, which is given by

$$\mathbf{E} = -k\frac{q}{r^2}\hat{\mathbf{r}} \tag{1}$$

$$= -k\frac{q}{a^2+4a^2}\hat{\mathbf{r}} \tag{2}$$

$$= \boxed{-\frac{kq}{5a^2}}\hat{\mathbf{r}}. \tag{3}$$

As the figure suggests, the simplest way to express the direction of $\hat{\mathbf{r}}$ is to orient an axis along the line between the unpaired charge and P, so that, for example, $\hat{\mathbf{r}} = \mathbf{i}$. Otherwise, the answer in component form is

$$\boxed{\mathbf{E} = \frac{8kq}{5\sqrt{5}a}\left(-2\mathbf{i}+\mathbf{j}\right)}, \tag{4}$$

and the acute angle the electric field at P makes with the x-axis is $\tan^{-1}(-0.5) = \boxed{26.6^\circ}$.

2 Conductors

At electrostatic equilibrium, a spherical conductor of radius r creates an electric field just outside its surface of magnitude E. What is the total charge on the sphere?

SOLUTION

The total charge is given by

$$Q = A\sigma \tag{5}$$

$$= AE\epsilon_0 \tag{6}$$

$$= \boxed{4\pi r^2 E\epsilon_0}. \tag{7}$$

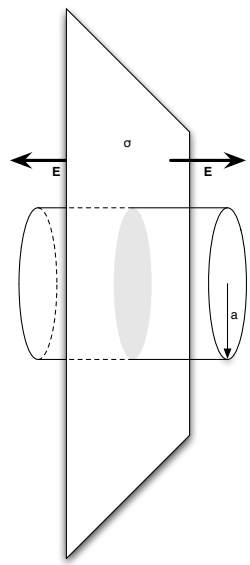
3 Gauss' Law

Using an appropriate Gaussian surface, find the electric field of an infinite plane carrying a uniform surface charge density σ.

SOLUTION

An appropriate* Gaussian surface is one made up of component surfaces at which the electric field lines are either (1) normal or (2) parallel to the normal vectors of the surface, or (3) of constant magnitude, including (4) zero. The component surfaces of a cylinder oriented as shown in the figure satisfies the first two of these criteria very nicely, although a cylinder is not the only possible choice. The only electric flux—due to the circular patch of enclosed charge—through the cylinder occurs at its two circular faces, where **E** is everywhere constant and parallel to the normal vectors. This allows the simplification of the inner product

*An "appropriate" Gaussian surface may be taken to mean one that effects the mathematically *convenient* application of Gauss' Law to the problem at hand.



in Gauss' Law, and further allows the removal of constant E from the integrand, thus

$$\oint \mathbf{E} \cdot a\mathbf{A} = \frac{q_{\text{enclosed}}}{\epsilon_0} \tag{8}$$

$$E \oint dA = \frac{\pi a^2 \sigma}{\epsilon_0} \tag{9}$$

$$E 2\cancel{\pi a^2} = \frac{\cancel{\pi a^2} \sigma}{\epsilon_0} \tag{10}$$

$$E = \boxed{\frac{\sigma}{2\epsilon_0}}. \tag{11}$$

4 Electric Potential

Show the mathematical connection between the *potential energy* of a charge-field system and the *electric potential* of the field. Explain the connection in words.

SOLUTION

The relevant expression is

$$\Delta V = \frac{\Delta U}{q_0}, \tag{12}$$

which states that the electric potential difference is simply the difference in the charge-field system's electrical potential energy divided by the charge.

5 Potential Difference

The positive terminal of a 9-V battery is connected to a large circular conducting plate of radius r. The negative terminal is connected to another identical plate. The plates are arranged parallel to each other, separated by a distance d. A charge q of mass m is released halfway between the plates. How fast is it going when it reaches the negative plate?

SOLUTION

Any kinetic energy possessed by the charge when it reaches the negative plate must have been gained from the field as the charge moved through a potential difference $\Delta V = -9/2 = -4.5$ V, thus

$$q\Delta V + \frac{1}{2}mv^2 = 0, \tag{13}$$

and

$$v = \sqrt{\frac{-2q\Delta V}{m}} \tag{14}$$

$$= \boxed{3\sqrt{\frac{q}{m}}}. \tag{15}$$