Some problems formatted like a final exam, each worth 2 points...

1 A conducting sphere of radius 12 cm produces an electric field $E = 2.3 \times 10^5$ N/C just outside its surface. The surface charge density on the sphere is

(a) 3.57 μC/m^2.

(b) 0.64 μC/m^2.

(c) 1.29 μC/m^2.

(d) 2.03 μC/m^2.

(e) None of the above.

SOLUTION

The electric field just outside a conductor is given by

$$E = \frac{\sigma}{\epsilon_0}, \tag{1}$$

thus

$$\begin{aligned} \sigma &= E\epsilon_0 && (2) \\ &= (2.3 \times 10^5)(8.85 \times 10^{-12}) && (3) \\ &= 2.03\ \mu\text{C/m}^2. && (4) \end{aligned}$$

2 When a positive charge moves in the direction of an electric field,

(a) the charge increases.

(b) the field does work on the charge.

(c) the charge decreases.

(d) the charge gains potential energy.

(e) the charge loses kinetic energy.

SOLUTION

A charges magnitude is unaffected by its movement through an electric field, so choices (a) and (c) are wrong. Positive charges move with *increasing* kinetic energy in the direction of electric field lines, which point towards lower potential, and thus lower potential energy, thus (d) and (e) are wrong. The field does in fact do work on the charge, choice (c).

3 An electron is fired directly toward the center of a large non-conducting plate that has excess surface charge density of $-2.0\ \mu\text{C/m}^2$. The initial energy of the electron is 1000 eV. If the electron is to stop just prior to reaching the plate, it must be fired from a distance of

(a) 2.25 mm.

(b) 4.43 mm.

(c) 6.63 mm.

(d) 8.85 mm.

(e) 22.1 mm.

SOLUTION

When the electron stops, it will have expended 1000 eV of kinetic energy which must be transferred to the charge-field system in the form of potential energy U. We use

$$\Delta U = -qEd, \tag{5}$$

and substitute for E the field due to an infinite plane of charge $E = \sigma/2\epsilon_0$ (to the electron, the insulator will certainly appear thus), and solving for d we have

$$\begin{aligned} d &= \left| -\frac{2\Delta U \epsilon_0}{q\sigma} \right| && (6) \\ &= \left| -\frac{2(10^3)\cancel{(1.6\times 10^{-19})}(8.85\times 10^{-12})}{\cancel{(-1.6\times 10^{-19})}(-2.0\times 10^{-6})} \right| && (7) \\ &= 8.85 \text{ mm.} && (8) \end{aligned}$$

4 Three point charges are located at the corners of an equilateral triangle of side length 0.20 m. $Q_2 = 3.0\,\mu\text{C}$ is at the origin, and $Q_3 = 3.0\,\mu\text{C}$ is along the x-axis. The net force on the -1.0-μC charge Q_1 due to the other two charges is closest to

(a) $-1.2\ \mathbf{j}$ N.

(b) $0.6\ \mathbf{i} - 1.2\ \mathbf{j}$ N.

(c) $-1.4\ \mathbf{j}$ N.

(d) $0.6\ \mathbf{i} + 1.0\ \mathbf{j}$ N.

(e) $-1.4\ \mathbf{i}$ N.

SOLUTION

From the symmetry of the problem, we know that no correct answer may contain an x-component, so choices (b), (d), and (e) are out. All that remains is to sum the y-components of the Coulombic forces on Q_1 due to each of the other two charges. The answer is (a).

5 Consider a linear charge density λ along the entire x-axis (that is, infinite). The electric potential at a point a distance h along the y-axis can be calculated by evaluating the integral

(a) $k_e \int_{-\infty}^{\infty} \frac{\lambda h x dx}{(x^2+h^2)^{3/2}}$

(b) $k_e \int_{-\infty}^{\infty} \frac{\lambda dx}{\sqrt{x^2+h^2}}$

(c) $k_e \int_{-\infty}^{\infty} \frac{\lambda h dx}{(x^2+h^2)^{3/2}}$

(d) $k_e \int_{-\infty}^{\infty} \frac{\lambda dx}{x^2+h^2}$

(e) $k_e \int_{-\infty}^{\infty} \frac{\lambda x dx}{\sqrt{x^2+h^2}}$

SOLUTION

Only choice (b) has the correct units of V=J/C.

...AND A COOL PROBLEM, WORTH 5 POINTS...

6 Remembering the mathematical relationship between potential and potential energy, find the amount of work required to assemble four identical charged particles of magnitude q at the corners of a square of side a.

SOLUTION

The complete expression for the energy is

$$
\begin{aligned}
U &= q_1V_2 + q_1V_3 + q_1V_4 + q_2V_3 + q_2V_4 + q_3V_4 && (9)\\
&= q_1\frac{kq_2}{r_{12}} + q_1\frac{kq_3}{r_{13}} + q_1\frac{kq_4}{r_{14}} + q_2\frac{kq_3}{r_{23}} + q_2\frac{kq_4}{r_{24}} + q_3\frac{kq_4}{r_{34}} && (10)\\
&= \frac{kq^2}{a} + \frac{kq^2}{a\sqrt{2}} + \frac{kq^2}{a} + \frac{kq^2}{a} + \frac{kq^2}{a\sqrt{2}} + \frac{kq^2}{a} && (11)\\
&= \frac{kq^2}{a}\left(4 + \frac{2}{\sqrt{2}}\right) && (12)\\
&= \boxed{5.4k\frac{q^2}{a}}. && (13)
\end{aligned}
$$