1. A ring of radius $a = 0.35$ m has a charge of $Q = 14.0 \times 10^{-6}$ C uniformly distributed on its surface. The center of the ring is at the origin. The electric field at a point P on the axis of the ring a distance $x = 1.50$ m from its center is closest to

   (a) $8.9 \times 10^4\ \mathbf{i}$ N/C
   (b) $1.1 \times 10^4\ \mathbf{j}$ N/C
   (c) $5.2 \times 10^4\ \mathbf{i}$ N/C
   (d) $2.7 \times 10^4\ \mathbf{i}$ N/C
   (e) $(2.7 \times 10^4\ \mathbf{i} + 2.7 \times 10^4\ \mathbf{j})$ N/C

   Solution

   If we do not recall the formula for the electric field on the axis of a ring of charge, we may find it by integrating. But simply, we use

$$\mathbf{E} = \frac{kx}{(a^2 + x^2)^{3/2}} Q\,\mathbf{i},$$

   where x is the distance from the plane of the ring, a is the ring's radius, and Q is the total charge on the ring. Thus,

$$\begin{aligned}\mathbf{E} &= \frac{(9 \times 10^9)(1.5)}{[(0.35)^2 + (1.5)^2]^{3/2}}(1.4 \times 10^{-6})\,\mathbf{i} \\ &= \boxed{5.2 \times 10^4\,\mathbf{i}\,\mathrm{N/C}}.\end{aligned}$$

2. Two point charges are located on the x-axis: a 2.0 μC charge at the origin and a 3.0 μC charge 5 cm away in the positive direction. The total electric field at a point P between the other two on the x-axis, 2 cm away from the origin, is closest to

   (a) $7.5 \times 10^7\ \mathbf{i}$ N/C
   (b) $1.5 \times 10^7\ \mathbf{j}$ N/C
   (c) $-7.5 \times 10^7\ \mathbf{i}$ N/C
   (d) $-1.5 \times 10^7\ \mathbf{i}$ N/C
   (e) $1.5 \times 10^7\ \mathbf{i}$ N/C

   Solution

   The principle of superposition allows the simple addition of the electric fields due to each respective source charge, thus

$$\begin{aligned}
\mathbf{E} &= \mathbf{E}_1 + \mathbf{E}_2 \\
&= k\left(\frac{q_1}{r_1^2} - \frac{q_2}{r_2}\right)\mathbf{i} \\
&= (9\times 10^9)\left[\frac{2.0\times 10^{-6}}{(0.02)^2} - \frac{3.0\times 10^{-6}}{(0.03)^2}\right]\mathbf{i} \\
&= \boxed{1.5\times 10^7\,\mathbf{i}\,\mathrm{N/C}}.
\end{aligned}$$

3. The electric field between two surfaces is given by $\vec{E} = (500r^2)\hat{\mathbf{r}}$ where $\vec{E}$ is in V/m and r is in m. As you move along in the direction of the electric field from $r = 1.0$ m to $r = 5.0$ m, the magnitude of the potential difference you move through is closest to

   (a) 12 000 V
   (b) 21 000 V
   (c) 14 000 V
   (d) 4 000 V
   (e) 8 000 V

   Solution

   The potential difference is given by

$$\begin{aligned}
\Delta V &= -\int_{\mathsf{a}}^{\mathsf{b}} \mathbf{E}\cdot d\mathbf{s} \\
&= -500\int_1^5 r^2\hat{\mathbf{r}}\cdot d\mathbf{s} \\
&= -500\int_1^5 r^2 dr \\
&= -\frac{500}{3}\left[r^3\right]_1^5 \\
&\approx \boxed{21\ 000\ \mathrm{V}}.
\end{aligned}$$

...MORE ON THE NEXT PAGE...

THREE OTHER PROBLEMS, WORTH 3 POINTS EACH...

4. Use Gauss' Law to find the electric field at a distance r away from and infinitely long line charge of uniform charge density λ. *Hint: use a cylinder!*

   Solution

   Heeding the hint, we construct a Gaussian cylinder of radius r and height h coaxial to the infinite line of charge. On the curved surface of the cylinder, the radially oriented electric field lines will everywhere be parallel to the normal and of constant magnitude. On the flat faces of the cylinder, the field lines will be perpendicular to the normal, and there will be no flux. Thus

$$\begin{aligned}\Phi = \oint \mathbf{E}\cdot d\mathbf{A} &= \frac{q_{\text{in}}}{\epsilon_0}\\ = \oint E dA &= \frac{h\lambda}{\epsilon_0}\\ = E\oint dA &= \frac{h\lambda}{\epsilon_0}\\ = E2\pi r h &= \frac{h\lambda}{\epsilon_0}\end{aligned}$$

   Solving for E gives

$$\boxed{\mathbf{E} = \frac{\lambda}{2\pi r\epsilon_0}\hat{\mathbf{r}}}.$$

5. What is the electric potential at a distance $r = 0.529\times 10^{-10}$ m from a proton? (This is the average distance between the proton and electron in a hydrogen atom.) What is the potential energy of the electron and the proton at this separation? Express your answer in electron volts (eV).

   Solution

   The equation for electrical potential energy is

$$\begin{aligned}U &= qV\\ &= qk\frac{q'}{r}\\ &= -ek\frac{e}{r}\\ &= -e\frac{(9\times 10^9)(1.6\times 10^{-19})}{0.529\times 10^{-10}}\\ &= \boxed{27\text{ eV}}.\end{aligned}$$

6. Find the electric field for the electric potential function $V(x) = 100 - 25x$. Where (for what value of x) is the potential zero? Write another electric potential function that corresponds to the same electric field with $V = 0$ at $x = 0$.

   Solution

   The electric field is the negative gradient of the potential, thus

$$\begin{aligned}\mathbf{E} &= -\nabla V \\ &= -\frac{\partial V}{\partial x}\mathbf{i} + -\frac{\partial V}{\partial y}\mathbf{j} + -\frac{\partial V}{\partial z}\mathbf{k} \\ &= -25\mathbf{i} + 0 + 0 \\ &= \boxed{-25\mathbf{i}}.\end{aligned}$$

   Solving the potential function for x at $V = 0$, we have

$$\boxed{x = \frac{100}{25} = 4}. \tag{1}$$

   For any d, the potential function $\boxed{V(x) = d - 25x}$ will correspond to the same electric field.