Six problems from Serway...make sure you do the problems, not the questions!

Due Wednesday, 6 February 2008.

1 Serway, Problem 26.4

SOLUTION

Since we are treating the cloud-earth system as a parallel plate capacitor, we may use

$$C = \frac{\epsilon_0 A}{d} \tag{1}$$

$$= \frac{(8.9 \times 10^{-12})(10^6)}{800} \tag{2}$$

$$= \boxed{11 \text{ nF}}. \tag{3}$$

Continuing to treat the cloud as a conducting capacitor plate, from the given dielectric strength of the air we can determine

$$q = \sigma A \tag{4}$$

$$= E_{\mathsf{max}} \epsilon_0 A \tag{5}$$

$$= (3 \times 10^6)(8.9 \times 10^{-12})(800) \tag{6}$$

$$= \boxed{26.7 \text{ C}}. \tag{7}$$

2 Serway, Problem 26.11

SOLUTION

For a spherical capacitor with inner radius a and outer radius b we have

$$C = \frac{ab}{k_{\mathsf{e}}(b-a} \tag{8}$$

$$= \frac{(0.07)(0.14)}{(9\times 10^9)(0.14-0.07)} \tag{9}$$

$$= \boxed{15.6\ \text{pF}}.$$

The potential difference between spheres is given by

$$\Delta V = \frac{Q}{C} \tag{10}$$

$$= \frac{4\times 10^{-6}}{1.56\times 10^{-11}} \tag{11}$$

$$= \boxed{256\ \text{kV}}. \tag{12}$$

3 Serway, Problem 26.19

SOLUTION

The initial charge Q_1 on C_1 is

$$Q_1 = C_1 \Delta V_1 \tag{13}$$

$$= (6\times 10^{-6})(20) \tag{14}$$

$$= \boxed{120\ \mu\text{C}}. \tag{15}$$

The charge then distributes between C_1 and C_2 according to

$$Q_1' + Q_2' = 1.2\times 10^{-4} \tag{16}$$

and

$$\Delta V' = \frac{Q_1'}{C_1} = \frac{Q_2'}{C_2}. \tag{17}$$

Substituting, we have

$$Q_1' = 1.2 \times 10^{-4} - \frac{C_2}{C_1} Q_1' \tag{18}$$

$$= \frac{1.2 \times 10^{-4}}{1 + \dfrac{C_2}{C_1}} \tag{19}$$

$$= \frac{1.2 \times 10^{-4}}{1 + 5\frac{1}{2}} \tag{20}$$

$$= \boxed{80\ \mu\text{C}}. \tag{21}$$

And therefore, $Q_2' = 40\ \mu$C.

4 Serway, Problem 26.23

The equivalent capacitance C_{eq} is given by

$$C_{\text{eq}} = \frac{\left(\dfrac{2C_1C_2}{C_1 + C_2} + C_3\right) 2C_2}{\dfrac{2C_1C_2}{C_1 + C_2} + C_3 + 2C_2} \tag{22}$$

$$= \frac{\left(\dfrac{2 \times 5 \times 10}{5 + 10} + 2\right) 2 \times 10}{\dfrac{2 \times 5 \times 10}{5 + 10} + 2 + 2 \times 10} \tag{23}$$

$$= \boxed{6.05\ \mu\text{F}}. \tag{24}$$

5 Serway, Problem 26.29

Using Serway's hint, we write $W = \int F dx$, or

$$F = \frac{dW}{dx} \tag{25}$$

$$= \frac{dU}{dx} \tag{26}$$

$$= \frac{d}{dx}\frac{Q^2}{2C} \tag{27}$$

$$= \frac{d}{dx}\frac{xQ^2}{2A\epsilon_0} \tag{28}$$

$$= \frac{Q^2}{2A\epsilon_0}. \tag{29}$$

6 Serway, Problem 26.33

The capacitance of a spherical conductor of radius R is

$$C = \frac{Q}{V} = \frac{\cancel{Q}R}{k_{\mathsf{e}}\cancel{Q}} = \frac{R}{k_{\mathsf{e}}}. \tag{30}$$

Plugging this into the expression for the energy stored in a capacitor, we have

$$U = \frac{Q^2}{2C} = \frac{Q^2 R}{2k_{\mathsf{e}}}. \tag{31}$$