These are due Friday, 7 March.

1. A point charge of magnitude $q = 4.5$ nC is moving with speed $v = 3.6 \times 10^7$ m/s parallel to the x axis along the line $y = 3$ m. Find the magnetic field at the origin produced by this charge when the charge is at the point $x = -4$ m, $y = 3$ m.

   Solution

   The magnetic field of the moving point charge is given by

$$\begin{aligned}
\mathbf{B} &= \frac{\mu_0}{4\pi}\frac{q\mathbf{v} \times \hat{\mathbf{r}}}{r^2} \\
&= -\frac{\mu_0}{4\pi}q\frac{v\sin\theta}{r^2}\mathbf{k} \\
&= -\frac{\mu_0}{4\pi}q\frac{3v}{5r^2}\mathbf{k} \\
&= -(10^{-7})(4.5 \times 10^{-9})\frac{3(3.7 \times 10^7)}{5(5)^2}\mathbf{k} \\
&= \boxed{-4 \times 10^{-10}\ \mathrm{T}\ \mathbf{k}}.
\end{aligned}$$

2. A circular loop of radius 5.0 cm has 12 turns and lies in the yz plane. It carries a current of 4 A in the direction such that the magnetic moment of the loop is along the x axis. Find the magnetic field on the x axis at

   (a) $x = 0$;

   (b) $x = 15$ cm;

   (c) $x = 3$ m.

   Solution

   We have three expressions for the magnetic field on the axis of a current-carrying loop of radius a, depending on how far we are from the plane of the loop. Using each in turn, as appropriate, we have:

   (a) For $x = 0$,

$$\begin{aligned}
B_x &= \frac{\mu_0 NI}{2a} \\
&= \frac{(4\pi \times 10^{-7})(12)(4)}{2(0.05)} \\
&= \boxed{6 \times 10^{-4}\ \mathrm{T}};
\end{aligned}$$

(b) For $x = 15$ cm,

$$\begin{aligned} B_x &= \frac{\mu_0 N I a^2}{2(a^2+x^2)^{3/2}} \\ &= \frac{(4\pi\times10^{-7})(12)(4)(0.05)^2}{2[(0.05)^2+(0.15)^2]^{3/2}} \\ &= \boxed{1.9\times10^{-5}\ \mathrm{T}}; \end{aligned}$$

(c) For $x = 3$ m,

$$\begin{aligned} B_x &= \frac{\mu_0}{2\pi}\frac{\mu}{x^3} \\ &= \frac{\mu_0}{2\pi}\frac{NI\pi a^2}{x^3} \\ &= (2\times10^{-7})\frac{(12)(4)\pi(0.05)^2}{3^3} \\ &= \boxed{2.8\times10^{-9}\ \mathrm{T}}. \end{aligned}$$

3. A small bar magnet of magnetic moment $\mu = 0.03\ \mathrm{A\cdot m^2}$ is placed at the center of the loop in the previous problem. The bar magnet's magnetic moment lies in the xy plane and makes an angle of 30° with the x axis. Neglecting any variation in B over the region of the magnet, find the torque on the magnet.

   Solution

   The torque on a magnetic moment is given by

$$\begin{aligned} \boldsymbol{\tau} &= \boldsymbol{\mu}\times\mathbf{B} \\ &= -\mu B\sin\theta\,\mathbf{k} \\ &= -(0.03)(6\times10^{-4})\sin30^\circ\,\mathbf{k} \\ &= \boxed{-9\times10^{-6}\ \mathrm{N\cdot m}\,\mathbf{k}}. \end{aligned}$$

4. Find the magnetic field at the center of a solenoid of length 20 cm, radius 1.4 cm, and 600 turns that carries a current of 4 A.

   Solution

The approximate solution is

$$\begin{aligned} B_x &= \mu_0 n I \\ &= \frac{\mu_0 N I}{l} \\ &= \frac{(4\pi \times 10^{-7})(600)(4)}{0.2} \\ &= \boxed{15 \text{ mT}}. \end{aligned}$$

5. Find the magnetic field at the center of a square current loop of side $L = 50$ cm carrying a current of 1.5 A.

   Solution

   The total field is four times that of any given side, thus

$$\begin{aligned} B &= 4B_{\text{side}} \\ &= 4\frac{\mu_0}{4\pi}\frac{2I}{L}(\sin\theta_1 + \sin\theta_2) \\ &= 4 \times 10^{-7}\frac{2(1.5)}{0.5}(\sin 45^\circ + \sin 45^\circ) \\ &= \boxed{3.4\ \mu\text{T}}. \end{aligned}$$

6. A long, straight wire carrying a current of 1.7 A in the positive z direction lies along the line $x = -3$ cm, $y = 0$. A similar wire carrying a current of 1.7 A in the positive z direction lies along the line $x = +3$ cm, $y = 0$. Find the magnetic field at a point P on the y axis at $y = 6$ cm.

   Solution

   The field at P is the vector sum of magnetic fields $\mathbf{B}_{\mathsf{L}}$ and $\mathbf{B}_{\mathsf{R}}$, due to the left and right

wires respectively, thus

$$\begin{aligned}
\mathbf{B} &= \mathbf{B}_{\mathsf{L}} + \mathbf{B}_{\mathsf{R}} \\
&= -2B_{\mathsf{L}} \cos\theta \, \mathbf{i} \\
&= -2\frac{\mu_0 I}{2\pi R} \cos\theta \, \mathbf{i} \\
&= -\frac{\mu_0 I y}{\pi R^2} \, \mathbf{i} \\
&= -\frac{\mu_0 I y}{\pi(x^2+y^2)} \, \mathbf{i} \\
&= -\frac{(4\pi \times 10^{-7})(1.7)(0.06)}{\pi[(0.03)^2+(0.06)^2]} \, \mathbf{i} \\
&= \boxed{-9 \ \mu\mathrm{T} \ \mathbf{i}}.
\end{aligned}$$

7. A long, straight wire of radius R carries a current I that is uniformly distributed over the cross sectional area of the wire. Find the magnetic field both outside and inside the wire.

   Solution

   Applying Ampère's Law

$$\begin{aligned}
\mu_0 I &= \oint \mathbf{B} \cdot d\mathbf{s} \\
&= B \oint ds \\
&= B 2\pi r
\end{aligned}$$

   we find

$$B = \frac{\mu_0 I}{2\pi r}$$

   Treating each case:

   (a) for outside the wire

$$B = \boxed{\frac{\mu_0 I}{2\pi r}};$$

(b) for inside the wire

$$\begin{aligned} B &= \frac{\mu_0 (r^2/R^2) I}{2\pi r} \\ &= \boxed{\frac{\mu_0 I r}{2\pi R^2}}. \end{aligned}$$