

NAME: Key

ALPHA \_\_\_\_\_

This exam consists of three parts: Part I is multiple choice (48%), Part II is short-answer problems (30%), and Part III is long answer (22%). Each problem from Part I is 8 points, from Part II is 6 points, from Part III is 15 points (out of 200 points total).

- Write your name, alpha, instructor, and section number on each of the three parts of the test and your Scantron bubble sheet. BUBBLE IN your ALPHA number.
- TI-36X Pro CALCULATORS ALLOWED. Calculators may not be shared.
- ALL communication devices (cell phones, smart watches, etc.) are prohibited and must be put away during the exam. If you need to leave the classroom you must leave all said devices in the classroom.

PART I: MULTIPLE CHOICE (40%). Do all work on the exam packet. Correct answers receive 5/5 points. Incorrect answers receive 0/5 points.

1. Which of the following functions is a solution to the differential equation  $\frac{dy}{dt} + \frac{y}{t} = 5$ ?

- 1 D
- $y = 5t + y \ln t$
  - $y = \frac{5}{2}t + C$
  - $y = 5t + C$
  - $y = \frac{5}{2}t + \frac{C}{t}$
  - $y = e^{5t + \ln t + C}$

Handwritten work for Q1:

$$\mu = e^{\int \frac{1}{t} dt} = e^{\ln t} = t$$

$$(ty)' = 5t$$

$$ty = \frac{5}{2}t^2 + C$$

$$y = \frac{5}{2}t + \frac{C}{t}$$

2.  $y = x^2$  is a solution to  $x^2y'' - 2xy' + ky = 0$  for  $k$  equal to

- 2 B
- 1
  - 2
  - 3
  - 4
  - 5

Handwritten work for Q2:

$$y' = 2x$$

$$y'' = 2$$

$$2 - 4 + k = 0$$

$$k = 2$$

$$2x^2 - 2x(2x) + kx^2 = 0$$

3. At the fixed point  $(\ln 3, 0)$ , the Jacobian matrix for the system  $\frac{dx}{dt} = y$ ,  $\frac{dy}{dt} = xe^x - 3x$  is:

- 3 E
- $\begin{bmatrix} 1 & 0 \\ 0 & e^x \end{bmatrix}$
  - $\begin{bmatrix} 0 & 1 \\ e^x & 0 \end{bmatrix}$
  - $\begin{bmatrix} 0 & 1 \\ \ln 3 & 0 \end{bmatrix}$
  - $\begin{bmatrix} 0 & 1 \\ 0 & 3 \ln 3 \end{bmatrix}$
  - $\begin{bmatrix} 0 & 1 \\ 3 \ln 3 & 0 \end{bmatrix}$

Handwritten Jacobian matrix:

$$DF = \begin{bmatrix} 0 & 1 \\ 3 \ln 3 & 0 \end{bmatrix}$$

Handwritten notes for Q3:

$\frac{\partial}{\partial x}$  derivative is  $1e^x + xe^x - 3$ , at  $x = \ln 3$

$$e^{\ln 3} + 3 \ln 3 - 3$$

4. A 1 kg mass is attached to a spring with constant 26 N/m. The system is immersed in a medium which has a damping force numerically equal to 2 times the instantaneous velocity. If  $x$  is the displacement of the mass from equilibrium, measured in meters, then

$$x'' + 2x' + 26x = 0.$$

Which of the following statements is true?

- 4A
- a)  $x(t) = c_1 e^{-t} \cos(5t) + c_2 e^{-t} \sin(5t)$ , and the system is underdamped.
  - b)  $x(t) = c_1 e^{-t} + c_2 e^{-5t}$ , and the system is overdamped.
  - c)  $x(t) = c_1 e^{-t} + c_2 e^{-5t}$ , and the system is underdamped.
  - d)  $x(t) = c_1 e^{-t} \cos(5t) + c_2 e^{-t} \sin(5t)$ , and the system is overdamped.
  - e)  $x(t) = c_1 t e^{-t} \cos(5t) + c_2 t e^{-t} \sin(5t)$ , and the system is critically damped.

$$r^2 + 2r + 26 = 0$$

$$(r+1)^2 + 25 = 0$$

$$-1 \pm 5i$$

from underdamped

5. The general solution for the separable differential equation  $\frac{dy}{dx} - 100x\sqrt{y} = 2\sqrt{y}$  is

- 5B
- a)  $y = e^{(100x+2)^2} + C$
  - b)  $y = (25x^2 + x + C)^2$
  - c)  $y = 50x^2 + 2x + C$
  - d)  $y = e^{200x+4+C}$
  - e)  $y = (e^{98x} + C)^2$

$$\frac{dy}{dx} = 2\sqrt{y} [1 + 50x]$$

$$\frac{1}{2\sqrt{y}} dy = (50x + 1) dx$$

$$\sqrt{y} = 25x^2 + x + C$$

6. Given that  $y = t^4$  and  $y = 1/t$  are linearly independent solutions of the homogeneous equation

$$t^2 y'' - 2ty' - 4y = 0,$$

and that  $y = t - 1$  is a particular solution of the non-homogeneous equation

$$t^2 y'' - 2ty' - 4y = 4 - 6t$$

which of the following is the general solution of the non-homogeneous equation?

- 6C
- a)  $y = c_1 t + c_2 + c_3 t^4 + c_4 \frac{1}{t}$
  - b)  $y = c_1 (t - 1) + c_2 t^4 + c_3 \frac{1}{t}$
  - c)  $y = t - 1 + c_1 t^4 + c_2 \frac{1}{t}$
  - d)  $y = c_1 t^4 + c_2 \frac{1}{t}$
  - e)  $y = c_1 t - c_2$ .

7. The matrix of the system  $\frac{dx}{dt} = x - y$   $\frac{dy}{dt} = 5x - y$  has eigenvalues  $\lambda = \pm 2i$ . For the eigenvalue  $2i$  an eigenvector is

$\begin{bmatrix} 1 \\ 1 - 2i \end{bmatrix}$ . Two fundamental solutions for the system are:

- 7A a)  $\begin{bmatrix} \cos(2t) \\ \cos(2t) + 2\sin(2t) \end{bmatrix}$  and  $\begin{bmatrix} \sin(2t) \\ -2\cos(2t) + \sin(2t) \end{bmatrix}$       b)  $\begin{bmatrix} \cos(2t) \\ \cos(2t) \end{bmatrix}$  and  $\begin{bmatrix} 0 \\ -\sin(2t) \end{bmatrix}$   
 c)  $\begin{bmatrix} \sin(2t) \\ \sin(2t) \end{bmatrix}$  and  $\begin{bmatrix} 0 \\ -\cos(2t) \end{bmatrix}$       d)  $\begin{bmatrix} \cos(2t) \\ \sin(2t) \end{bmatrix}$  and  $\begin{bmatrix} 0 \\ -\sin(2t) \end{bmatrix}$       e)  $\begin{bmatrix} \sin(2t) \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 0 \\ \cos(2t) \end{bmatrix}$

$$(\cos(2t) + i\sin(2t)) \begin{bmatrix} 1 \\ 1 - 2i \end{bmatrix} = \begin{bmatrix} \cos(2t) + i\sin(2t) \\ \cos(2t) - 2i\cos(2t) + i\sin(2t) + 2\sin(2t) \end{bmatrix}$$

extract the real and imaginary parts

8. Consider the system  $\frac{dx}{dt} = 5y^5$   $\frac{dy}{dt} = k(x+1)$ , where  $k$  is a constant. Find  $k$  such that the energy

$L(x, y) = \frac{1}{2}(x+1)^2 + \frac{1}{2}y^6$  is conserved along trajectories.

- a)  $k = \frac{5}{2}$     b)  $k = \frac{2}{5}$     c)  $k = -\frac{2}{15}$     d)  $k = -\frac{2}{5}$     e)  $k = -\frac{5}{3}$

8E

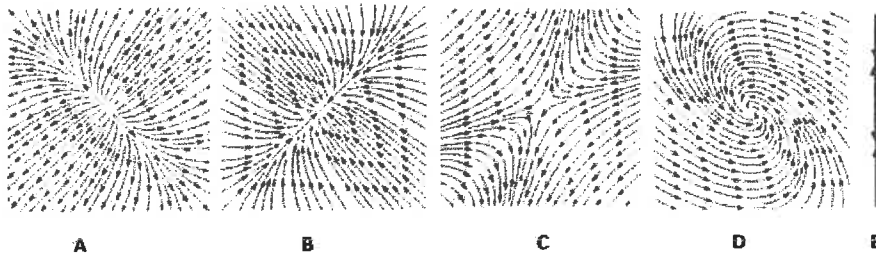
$$\frac{\partial L}{\partial x} = (x+1) \quad \frac{\partial L}{\partial y} = 3y^5$$

$$\frac{dL}{dt} = (x+1)5y^5 + 3y^5 k(x+1) = y^5(x+1) [5 + 3k] \quad k = -\frac{5}{3}$$

9. Which of the following phase portraits is that of the linear system  $\vec{X}' = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \vec{X}$ ?

- a) A    b) B    c) C    d) D    e) E

9B

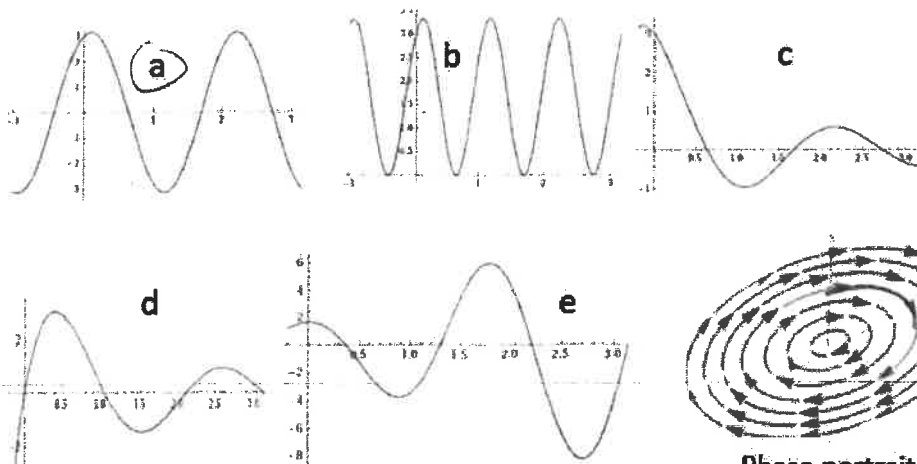


$$\begin{vmatrix} \lambda + 2 & -1 \\ -1 & \lambda + 2 \end{vmatrix} = (\lambda + 2)^2 - 1 = (\lambda + 3)(\lambda + 1) \quad \text{stable node} \quad B$$

$\lambda^2 + 4\lambda + 3$       Page 3

10. The phase portrait of a linear system is given below. Consider the  $y$  component of the solution whose initial condition is  $x(0) = 0, y(0) = y_0 > 0$ . Which of the following is the graph of  $y$  as a function of time?

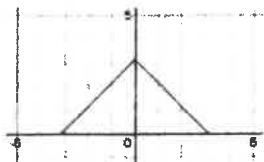
10A



Phase portrait  
periodic motion

11. Let  $f$  be the periodic function with half-period  $p = \pi$  given by  $f(x) = \begin{cases} \pi + x & \text{if } -\pi < x < 0 \\ \pi - x & \text{if } 0 < x < \pi \end{cases}$  whose graph is given below. The Fourier series of  $f$  equals

11D



- a)  $\frac{\pi}{2} + \frac{4 \sin(\pi x)}{\pi} + \frac{4 \sin(3\pi x)}{9\pi} + \frac{4 \sin(5\pi x)}{25\pi} + \dots$
- ~~b)  $-\frac{\pi}{2} + \frac{4 \cos(\pi x)}{\pi} + \frac{4 \cos(3\pi x)}{9\pi} + \frac{4 \cos(5\pi x)}{25\pi} + \dots$~~
- ~~c)  $-\frac{\pi}{2} + \frac{4 \sin(x)}{\pi} + \frac{4 \sin(3x)}{9\pi} + \frac{4 \sin(5x)}{25\pi} + \dots$~~
- d)  $\frac{\pi}{2} + \frac{4 \cos(x)}{\pi} + \frac{4 \cos(3x)}{9\pi} + \frac{4 \cos(5x)}{25\pi} + \dots$
- ~~e)  $-\frac{\pi}{2} + \frac{\sin(2x)}{4\pi} - \frac{\sin(4x)}{16\pi} + \frac{\sin(6x)}{36\pi} - \frac{\sin(8x)}{64\pi} + \dots$~~

$p = \pi$

$\frac{Df}{p} x = nx$ , even function

$a_0 = \frac{2}{\pi} \int_0^{\pi} (\pi - x) dx = \frac{-2}{\pi} \int_0^{\pi} (x - \pi) dx$

$a_0 = \frac{-2}{\pi} \left[ \frac{1}{2} (x - \pi)^2 \right]_0^{\pi}$

$a_0 = \frac{1}{\pi} \pi^2 = \pi$

$\frac{a_0}{2} = \frac{\pi}{2}$

12B

12. An undamped mass-spring system with mass of 25 kg and spring constant of 100 N/m is driven by a force  $F(t)$ . Which of the following functions  $F$  leads to resonance?

- a)  $100 \cos t$
- b)  $100 \cos(2t)$
- c)  $100 \sin(10t)$
- d)  $4 \cos(5t)$
- e)  $25 \sin(4.9t)$

$25x'' + 100x = F(t)$

$x'' + 4x = \frac{1}{25} F(t)$

$r^2 + 4 = 0, r = \pm 2i$

$\omega = 2$

4 p each

Part II: SHORT ANSWERS Show your work. You MUST write your *simplified* answer in the box.

13. The general solution for the differential equation  $x''' + 12x'' + 32x' = 0$  is

$$\begin{aligned} r^3 + 12r^2 + 32r &= 0 \\ r(r^2 + 12r + 32) &= 0 \\ r(r+4)(r+8) &= 0 \\ r &= 0, -4, -8 \end{aligned}$$

Simplified Answer:  $x(t) =$   
 $c_1 + c_2 e^{-4t} + c_3 e^{-8t}$

14. The general solution for the differential equation  $x'' + 12x' + 36x = 0$  is

$$\begin{aligned} r^2 + 12r + 36 &= 0 \\ (r+6)^2 &= 0 \end{aligned}$$

Simplified Answer:  $x(t) =$   
 $c_1 e^{-6t} + c_2 t e^{-6t}$

15. A particular solution for the differential equation  $x'' - 4x' + 3x = 30 \cos(3t)$  is  $x'' + x' = 30 \cos(3t)$

$$\begin{aligned} x &= A \sin(3t) + B \cos(3t) \\ x' &= 3A \cos(3t) - 3B \sin(3t) \\ x'' &= -9A \sin(3t) - 9B \cos(3t) \end{aligned} \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{add } x'' \text{ and } x'$$

$$(3A - 9B) \cos(3t) + (-3B - 9A) \sin(3t) = 30 \cos(3t) + 0 \sin(3t)$$

$3A - 9B = 30$	$A - 3B = 10$
$9A + 3B = 0$	$A - 3(-3A) = 10$
$B = -3A$	$A = 1$
	$B = -3$

Simplified Answer:  
 $x_p = \sin(3t) - 3 \cos(3t)$

16. The solution for the initial value problem  $x'' - 2x' + 5x = 0$  with  $x(0) = 0, y(0) = 6$  is

$$r^2 - 2r + 5 = 0 \quad (r-1)^2 + 4 = 0$$

$$r = 1 \pm 2i$$

$$x = C_1 e^t \cos(2t) + C_2 e^t \sin(2t)$$

$$x(0) = 0 \quad C_1 = 0$$

$$x = C_2 e^t \sin(2t)$$

$$x' = C_2 [e^t \sin(2t) + 2e^t \cos(2t)]$$

$$x'(0) = 2C_2 = 6 \quad C_2 = 3$$

Simplified Answer:

$$x = 3e^t \sin(2t)$$

17. Find all the solutions  $u = u(x, y)$  of product type for the partial differential equation  ~~$3u_x - u_y = 4u$~~   $3u_x - u_y = 4u$

$$u = X(x) Y(y)$$

$$3X'Y - XY' = 4XY, \div XY$$

$$3 \frac{X'}{X} = 4 + \frac{Y'}{Y} = \lambda$$

$$X' = 3\lambda X$$

$$X = e^{\frac{\lambda}{3}x} x_0$$

$$Y' = (\lambda - 4)Y$$

$$Y = e^{(\lambda - 4)y} y_0$$

$$C_0 = x_0 y_0 = \text{constant}$$

Simplified Answer:

$$u = C_0 e^{\frac{\lambda}{3}x + (\lambda - 4)y}$$

18. Find a particular solution for the system  $\vec{X}' = B\vec{X} + \vec{f}$  where  $B = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$  and  $\vec{f} = \begin{bmatrix} 0 \\ 24e^{4t} \end{bmatrix}$ . Use the fact that  $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$  is an eigenvector for  $B$  with  $\lambda = 4$  and that  $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$  is an eigenvector for  $\lambda = 0$ .

$$\phi = \begin{bmatrix} 2e^{4t} & -2 \\ e^{4t} & 1 \end{bmatrix} \quad \det \phi = 4e^{4t}$$

$$X_p = \phi u$$

$$\phi u' = \begin{bmatrix} 0 \\ 24e^{4t} \end{bmatrix}$$

$$\phi^{-1} = \frac{1}{4e^{4t}} \begin{bmatrix} 1 & 2 \\ -e^{4t} & 2e^{4t} \end{bmatrix}$$

$$\phi^{-1} = \begin{bmatrix} \frac{1}{4}e^{-4t} & \frac{2}{4}e^{-4t} \\ -\frac{1}{4} & \frac{2}{4} \end{bmatrix}$$

$$u' = \begin{bmatrix} \frac{1}{4}e^{-4t} & \frac{2}{4}e^{-4t} \\ -\frac{1}{4} & \frac{2}{4} \end{bmatrix} \begin{bmatrix} 0 \\ 24e^{4t} \end{bmatrix} = \begin{bmatrix} 48/4 \\ 48/4 e^{4t} \end{bmatrix} = \begin{bmatrix} 12 \\ 12e^{4t} \end{bmatrix}$$

$$u = \int u' dt$$

$$u = \begin{bmatrix} 12t \\ 3e^{4t} \end{bmatrix}$$

$$X_p = \begin{bmatrix} 2e^{4t} & -2 \\ e^{4t} & 1 \end{bmatrix} \begin{bmatrix} 12t \\ 3e^{4t} \end{bmatrix} = \begin{bmatrix} 24te^{4t} - 6e^{4t} \\ 12te^{4t} + 3e^{4t} \end{bmatrix} = e^{4t} \begin{bmatrix} 24t - 6 \\ 12t + 3 \end{bmatrix}$$

check  $\frac{d}{dt} [e^{4t} (24t - 6)] = 96te^{4t}$

$$x' = 96te^{4t}$$

$$2x + 4y = e^{4t} (48t - 12) + e^{4t} (48t + 12) = x'$$

$$y' = \frac{d}{dt} [e^{4t} (12t + 3)] = e^{4t} (48t + 24)$$

$$x + 2y + 24e^{4t} = e^{4t} [24t - 6 + 24t + 6 + 24] \quad \text{Page 7}$$

Simplified Answer:  $X_p =$

$$e^{4t} \begin{bmatrix} 24t - 6 \\ 12t + 3 \end{bmatrix}$$

19. The temperature  $u(x, t)$  on a thin rod satisfies the heat equation with zero ends condition

$$\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0$$

$$u(x, 0) = x(\pi - x), \quad 0 < x < \pi.$$

Use the fact that the half-range Fourier sine series of  $f(x) = x(\pi - x)$ ,  $0 < x < \pi$  is  $f(x) = \sum_{n=1}^{\infty} \frac{4[1 - (-1)^n]}{\pi n^3} \sin(nx)$  to give an approximation of  $u(x, t)$  that uses the first three nonzero terms from  $u$ .

Do NOT show the steps of the separation of variables process here. Use the provided formula for  $u$ .

$k=4 \quad L=\pi \quad u = \sum_{n=1}^{\infty} C_n e^{-4n^2 t} \sin(nx)$ 
 $C_n = \frac{4[1 - (-1)^n]}{\pi n^3}$ 
 $C_1 = \frac{8}{\pi}$   
 $-\frac{k n^2 L^2}{L^2} = -4n^2$ 
 $C_2 = 0 \quad C_3 = \frac{4(2)}{27\pi} \quad C_5 = \frac{4(2)}{125\pi}$   
 $\frac{n\pi}{L} x = nx$

**Simplified Answer:**

$$u \approx \frac{8}{\pi} e^{-4t} \sin(x) + \frac{2}{27\pi} e^{-36t} \sin(3x) + \frac{8}{125\pi} e^{-100t} \sin(5x)$$

20. Consider the function  $g$  given on its half period by  $g(x) = \begin{cases} 30 & \text{for } 0 < x \leq \pi/2, \\ 0 & \text{for } \pi/2 < x < \pi \end{cases}$

$\frac{n\pi}{P} x = \frac{n\pi}{\pi} x = nx$

Find the coefficient of  $\cos(nx)$  of the half-range cosine expansion of  $g$ .

$$a_n = \frac{2}{\pi} \int_0^{\pi} g(x) \cos(nx) dx = \frac{2}{\pi} \int_0^{\pi/2} 30 \cos(nx) dx = \frac{60}{\pi} \left. \frac{1}{n} \sin(nx) \right|_{x=0}^{x=\pi/2}$$

$$a_n = \frac{60}{n\pi} \sin\left(n \frac{\pi}{2}\right) = 0$$

**Simplified Answer:**

$$\frac{60}{n\pi} \sin\left(\frac{n\pi}{2}\right) = a_n$$

$$L_x = \frac{1}{2}x^3 - 2x$$

$$y = 2y \quad \left| \quad \frac{dL}{dt} = \frac{1}{2}x^3 - 2xy + 2xy - \frac{2y}{4}x^3 = 0 \right.$$

21. (14%) Consider the conservative system  $\begin{cases} x' = y \\ y' = x - \frac{1}{4}x^3 \end{cases}$  with conserved energy  $L(x, y) = \frac{1}{8}x^4 - x^2 + y^2$ .

a) The system has three fixed points, one being  $(-2, 0)$ . Find the remaining two fixed points. Denote these other points by  $M$  and  $N$ . All the questions that follow are about  $M$  and  $N$ ; none about  $(-2, 0)$ .

$$y = 0 \quad x - \frac{1}{4}x^3 = 0 \quad x = 0, \pm 2$$

$$x^3 - 4x = 0 \quad M(0,0) \quad N(2,0)$$

b) Find  $J_M$ , the Jacobian matrix of the system at  $M$ . Show a *detailed* phase portrait of the linear system  $X' = J_M X$ . Make sure to include the eigenvalues and all the elements used to produce the sketch.

$$J = \begin{bmatrix} 0 & 1 \\ 1 - \frac{3}{4}x^2 & 0 \end{bmatrix} \quad J_{00} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \lambda^2 - 1 = 0$$

$$\lambda = \pm 1$$

$$\lambda = 1 \quad \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \vec{v} = 0 \quad \vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda = -1 \quad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \vec{w} = 0 \quad \vec{w} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Saddle

c) Find  $J_N$ , the Jacobian matrix of the system at  $N$ . Show a *detailed* phase portrait of the linear system  $X' = J_N X$ . Make sure to include the eigenvalues and all the elements used to produce the sketch.

$$J_N = \begin{bmatrix} 0 & 1 \\ 1 - 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} \quad \lambda^2 + 2 = 0$$

$$\lambda = \pm \sqrt{2}i$$

$$\begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

test direction for  $J_N$  at  $(1,0)$

d) Calculate the Hessian  $H(x, y)$  of  $L$  and evaluate it at each of the points  $M$  and  $N$  in order to determine if the critical points  $M, N$  are local extrema or saddle points for  $L$ . (show work on the next page)

$$L = \frac{1}{8}x^4 - x^2 + y^2$$

$$L_x = \frac{1}{2}x^3 - 2x \quad L_{xx} = \frac{3}{2}x^2 - 2$$

$$L_{xy} = 0 \quad L_{yy} = 2$$

$$L_y = 2y$$

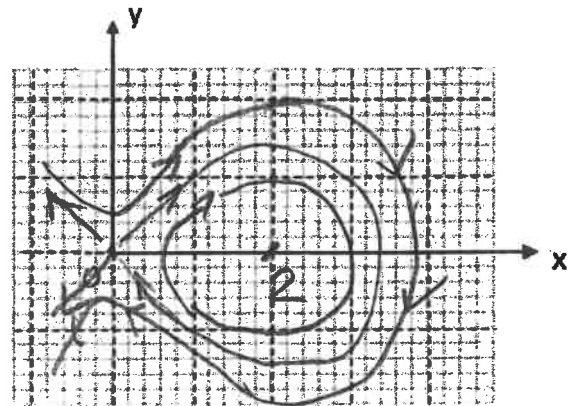
$$H = \begin{bmatrix} \frac{3}{2}x^2 - 2 & 0 \\ 0 & 2 \end{bmatrix}$$

at  $(0,0)$   $D = -4$  Saddle

at  $(2,0)$   $D = 8$  min/max

$$H(x, y) = \begin{bmatrix} \frac{3}{2}x^2 - 2 & 0 \\ 0 & 2 \end{bmatrix}$$

e) Sketch the phase portrait of the conservative system on the coordinate system below (the region  $-1 \leq x \leq 5$  and  $-2 \leq y \leq 2$ , which includes  $M$  and  $N$ , but does not include  $(-2, 0)$ ). Explain why the trajectory curves are as indicated near  $M$  and  $N$ .



22. (8%) Consider the heat equation  $\frac{\partial u}{\partial t} = 8 \frac{\partial^2 u}{\partial x^2}$  with zero ends, for a rod of length  $L = 4$ .

Find the general solution using separation of variables; use summation notation in your final answer. **You must show your work**, with one exception: you do NOT have to include the case of non-working exponentials in the  $x$  variable.

$$u = X(x)T(t)$$

Main Steps here:

$$T'X = 8TX'' \quad \leftarrow \quad |$$

$$\frac{T'}{T} = -8\omega^2$$

$$\text{and } \frac{X''}{X} = -\omega^2$$

$$X = \sin(\omega x)$$

$$4\omega = \pi, 2\pi, 3\pi, \dots, n\pi$$

$$\omega = n \frac{\pi}{4}$$

$$X(0) = X(4) = 0$$

$$\frac{T'}{T} = -8 \frac{n^2 \pi^2}{16} = -\frac{1}{2} n^2 \pi^2, \quad T = \exp\left(-\frac{1}{2} n^2 \pi^2 t\right)$$

$$u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{1}{2} n^2 \pi^2 t} \sin\left(n \frac{\pi}{4} x\right)$$

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