

Name: _____

Alpha: _____

Precalculus SM005 Fall 2017-2018

Final Exam

December 12, 2017

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General Instructions:

- You will have 3 hours to complete this exam. The exam contains a total of 220 points.
- Immediately fill out the top of the Scantron form using a number 2 pencil:
 - Bubble in your alpha.
 - Do not bubble in a version code.
 - Write your name and section.
- Read through the entire exam first. Start with the easy problems. You may complete the exam in any order.
- Ask questions if you do not understand the problem.
- No calculators, notes, or other resources are allowed. You must work alone.

Multiple Choice:

- There are 30 multiple choice questions, Questions 1 through 30.
- Each question is worth 4 points. The exam contains a total of 120 multiple choice points.
- You are required to complete all 30 questions.
- Record all multiple choice answers on the Scantron form.
- There is no partial credit for these questions. You will not be given any credit for work or answers in the test booklet.

Free Response:

- There are 17 free response questions, Questions 31 through 47.
- Questions 31 through 33 are worth 10 points each. Questions 34 through 47 are worth 5 points each. The exam contains a total of 100 free response points.
- You are required to complete all 17 questions.
- Show all your work in the test booklet.

- Partial credit may be awarded for these questions.

1. (4 points) In which quadrant(s) is $\sec(x)$ negative?
 - A. III and IV
 - B. II and III
 - C. I and III
 - D. I and IV
 - E. II only

2. (4 points) Determine $(0, 4] \cap [-2, 1]$. Give your answer in interval notation.
 - A. $[0, 1]$
 - B. $[-2, 0) \cup (0, 1) \cup (1, 4]$
 - C. $(0, 1)$
 - D. $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$
 - E. $[-2, 4]$

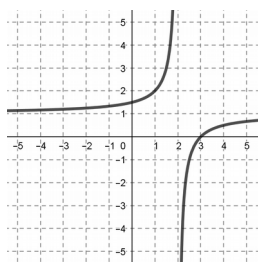
3. (4 points) Determine the values of sine and cosine of $\frac{5}{6}\pi$.
 - A. $\sin\left(\frac{5}{6}\pi\right) = \frac{1}{2}, \cos\left(\frac{5}{6}\pi\right) = \frac{\sqrt{3}}{2}$
 - B. $\sin\left(\frac{5}{6}\pi\right) = \frac{1}{2}, \cos\left(\frac{5}{6}\pi\right) = \frac{-\sqrt{3}}{2}$
 - C. $\sin\left(\frac{5}{6}\pi\right) = \frac{\sqrt{3}}{2}, \cos\left(\frac{5}{6}\pi\right) = \frac{-1}{2}$
 - D. $\sin\left(\frac{5}{6}\pi\right) = \frac{\sqrt{3}}{2}, \cos\left(\frac{5}{6}\pi\right) = \frac{1}{2}$
 - E. $\sin\left(\frac{5}{6}\pi\right) = \frac{-1}{2}, \cos\left(\frac{5}{6}\pi\right) = \frac{\sqrt{3}}{2}$

4. (4 points) Let $f(x) = x^2 - 3x + 4$. Evaluate $f(x+1)$.
 - A. $x^2 - x + 2$
 - B. $x^2 - 3x + 5$
 - C. $x^2 - x + 6$
 - D. $x^2 - x + 8$

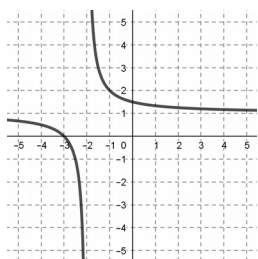
E. $x^2 - 3x + 2$

5. (4 points) Which of the following is a graph of the function $f(x) = \frac{1}{x-2} + 1$?

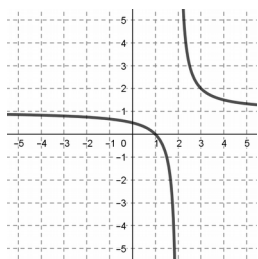
A.



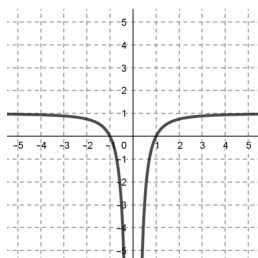
B.



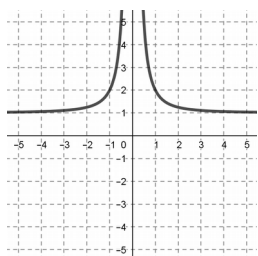
C.



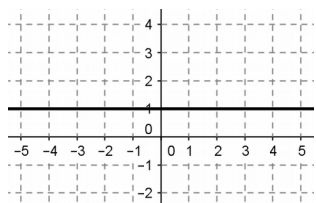
D.



E.



6. (4 points) What is the equation of the line shown in the graph below?



- A. $y=1$
B. $y=x$
C. $y=x+1$
D. $x=1$
E. $y^2=1$
7. (4 points) The expression $\ln(e) - \ln(x^3)$ is equivalent to
- A. $-\ln(x^3)$
B. $\ln(e) + \ln(-x^3)$
C. $\frac{1}{\ln(x^3)}$
D. $\frac{\ln(e)}{\ln(x^3)}$
E. $1 - \ln(x^3)$
8. (4 points) How many **real** solutions exist for the equation $4x^2 - 3x + 1 = 0$?
- A. 0, because the equation cannot be factored using whole numbers
B. 0, because the equation cannot be factored using real numbers
C. 1, because 1 and 4 are perfect squares
D. 2, because the degree of the polynomial is 2
E. cannot be determined, because the equation cannot be factored using whole numbers
9. (4 points) Determine the domain of $f(x) = \frac{\sqrt{3-x}}{x+1}$. Give your answer in interval notation.
- A. $(-1, 3]$
B. $(-1, \infty)$
C. $(-\infty, -1) \cup (-1, 3]$

D. $(-\infty, -1] \cup [-1, 3)$

E. $(-\infty, -1) \cup (-1, \infty)$

10. (4 points) Determine $\cot(\pi)$.
- A. -1
 - B. 0
 - C. 1
 - D. π
 - E. undefined
11. (4 points) The expression $\log_3(81x^2)$ is equivalent to
- A. $2\log_3(81x)$
 - B. $4+2\log_3(x)$
 - C. $4\log_3(x^2)$
 - D. $8\log_3(x)$
 - E. $4+x^2$
12. (4 points) Let $f(x)=\sqrt[3]{x}+2$. Determine $f^{-1}(x)$.
- A. x^3+2
 - B. $(x-2)^3$
 - C. $(y-2)^3$
 - D. x^3-2
 - E. $(x+2)^3$
13. (4 points) Determine $\sin^{-1}\left(\frac{-1}{2}\right)$. Give your answer in radians.
- A. $\frac{-1}{6}\pi$
 - B. $\frac{1}{6}\pi$
 - C. $\frac{-1}{3}\pi$
 - D. $\frac{1}{3}\pi$

E. $\frac{11}{6}\pi$

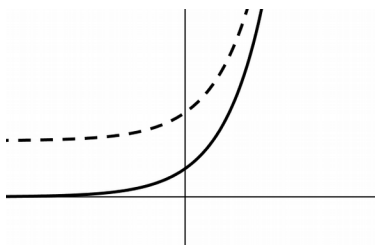
14. (4 points) Simplify $\frac{\sqrt{72}}{\sqrt{4x^6}}$ completely.
- A. $\frac{3\sqrt{2}}{\sqrt{x^6}}$
- B. $\sqrt{18}x^3$
- C. $\frac{3\sqrt{2}}{x^3}$
- D. $\frac{\sqrt{72}}{2x^6}$
- E. $3x^3\sqrt{2}$
15. (4 points) Let $(f \circ g)(x) = \sqrt{x^2 + x}$. $(f \circ g)(x)$ can be decomposed into
- A. $f(x) = \sqrt{x^2}, g(x) = \sqrt{x}$
- B. $f(x) = \sqrt{x}, g(x) = x^2 + x$
- C. $f(x) = \sqrt{x+1}, g(x) = \sqrt{x}$
- D. $f(x) = x^2 + x, g(x) = \sqrt{x}$
- E. $f(x) = \sqrt{x^2 + 1}, g(x) = x$
16. (4 points) In which quadrant(s) is the angle $\frac{-25}{4}\pi$ located?
- A. I
- B. II
- C. III
- D. IV
- E. $\frac{-25}{4}\pi$ could be located in more than one quadrant
17. (4 points) Determine $\tan^{-1}(0)$.
- A. $-\pi$
- B. 0
- C. $\frac{1}{2}\pi$

D. π

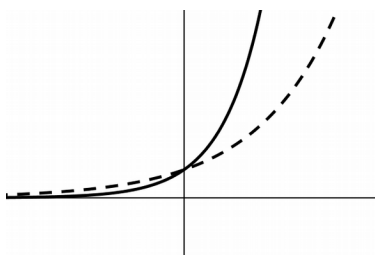
E. 2π

18. (4 points) The graph of 2^{2x} is a transformation of the graph of 2^x . The solid line represents 2^x . In which of the following graphs does the dotted line represent 2^{2x} ?

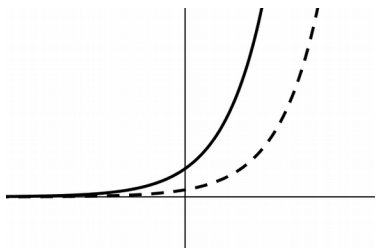
A.



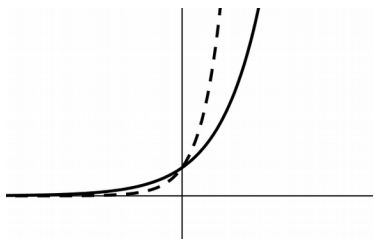
B.



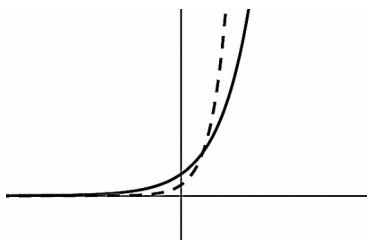
C.



D.

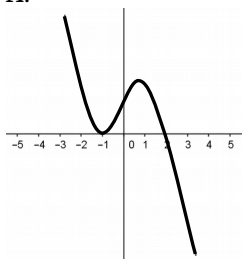


E.

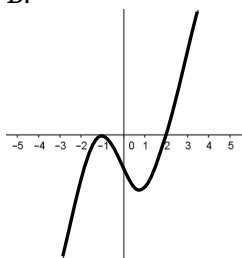


19. (4 points) Which of the following is a possible graph of $f(x) = -3(x+1)^2(x-2)$?

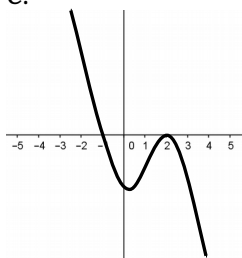
A.



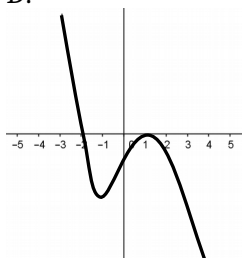
B.



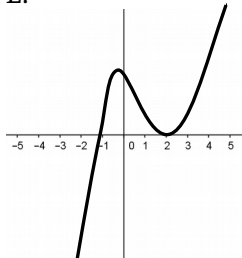
C.



D.



E.



20. (4 points) Simplify $\frac{3+3\sqrt{6}}{6}$ completely.

A. $\frac{1+\sqrt{2}}{2}$

B. $\frac{\sqrt{6}}{2}$

C. $\frac{1+\sqrt{6}}{2}$

D. $\frac{3\sqrt{6}}{2}$

E. $\frac{1+3\sqrt{6}}{2}$

21. (4 points) Calculate $\frac{22}{21} - \frac{18}{35}$. Simplify your answer completely.

A. $\frac{56}{105}$

B. $\frac{4}{105}$

C. $\frac{8}{15}$

D. $\frac{392}{735}$

E. $\frac{-2}{7}$

22. (4 points) Simplify $\log_a(a^x)$.

A. a^x

B. x^a

C. $\ln(x)$

D. x

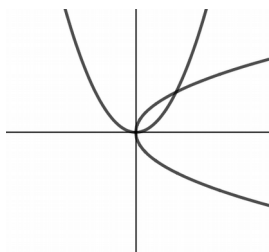
E. $x \cdot a$

23. (4 points) In which quadrant(s) is the angle $\frac{17}{6}\pi$ located?

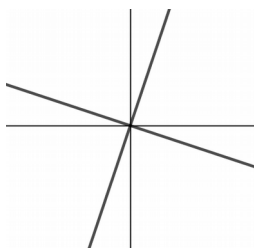
- A. I
- B. II
- C. III
- D. IV
- E. $\frac{17}{6}\pi$ could be located in more than one quadrant

24. (4 points) Which of the following pairs of graphs is a function and its inverse function?

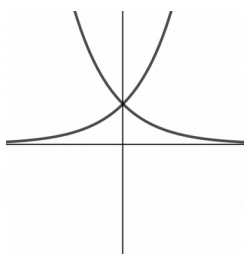
A.



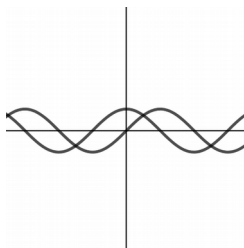
B.



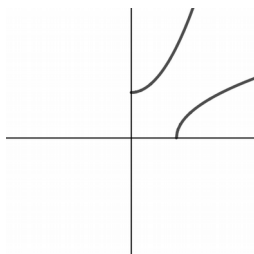
C.



D.



E.

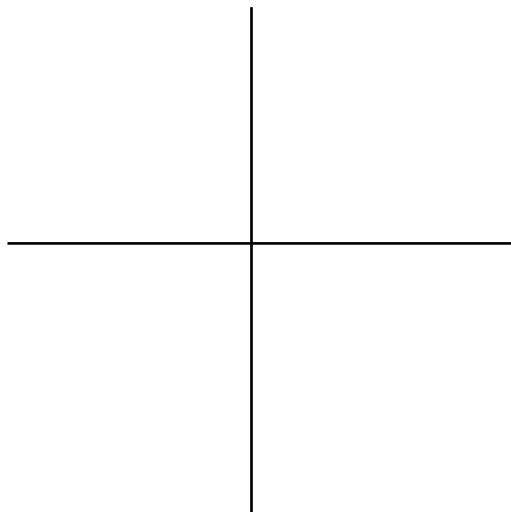


25. (4 points) A right triangle is composed of the origin, the x axis, and the point $(2, -1)$. Find the sine of the angle at the origin.
- A. $\frac{\sqrt{5}}{5}$
B. $-\sqrt{5}$
C. $\frac{-\sqrt{5}}{5}$
D. $\frac{2\sqrt{5}}{5}$
E. $\frac{-\sqrt{5}}{2}$
26. (4 points) Determine the equation of the line containing the points $(1, 4)$ and $(-3, 7)$. Give your answer in slope intercept form.
- A. $y = \frac{-4}{3}x + \frac{16}{3}$
B. $y = \frac{-4}{3}x + 4$
C. $y = \frac{-4}{3}x + 7$
D. $y = \frac{-3}{4}x + \frac{19}{4}$
E. $y = \frac{-3}{4}x + 3$
27. (4 points) Determine the domain of $f(x) = \ln(2 - x)$. Give your answer in interval notation.
- A. $(-\infty, 2) \cup (2, \infty)$
B. $(-\infty, 2)$
C. $(2, \infty)$
D. $(-\infty, 2]$
E. $[-2, 2]$

28. (4 points) Find the distance between the point $(2, 5)$ and the point $(3, -1)$.
- A. $\sqrt{37}$
 - B. $\sqrt{5}$
 - C. $\sqrt{41}$
 - D. 3
 - E. 7
29. (4 points) Which of the following represents the characteristics of $-3 \sin\left(\frac{1}{2}x\right)$?
- A. amplitude 3, period 4π , phase shift 0
 - B. amplitude -3 , period 4π , phase shift 0
 - C. amplitude 3, period 2π , phase shift $\frac{1}{2}\pi$
 - D. amplitude 1, period 2π , phase shift 0
 - E. amplitude 1, period π , phase shift $\frac{1}{2}\pi$
30. (4 points) Which of the following are true statements about e^x and $\ln(x)$?
- A. $e^1=0, \ln(0)=1$
 - B. $e^1=0, \ln(1)=0$
 - C. $e^0=1, \ln(1)=e$
 - D. $e^0=1, \ln(0)=e$
 - E. $e^0=1, \ln(1)=0$

31. (10 points) Solve $2x^2 + 5x = 12$ using two different methods. Simplify your answer(s) completely. Consider real solutions only.

32. (10 points) Sketch the graphs of e^x and $\ln(x)$ on the same set of axes. State the coordinates of any intercepts of each, the domain and range of each, and the **equations** of any asymptotes of each.



e^x coordinates of intercept(s): _____

e^x domain: _____

e^x range: _____

e^x **equation(s)** of asymptote(s): _____

$\ln(x)$ coordinates of intercept(s): _____

$\ln(x)$ domain: _____

$\ln(x)$ range: _____

$\ln(x)$ **equation(s)** of asymptote(s): _____

33. (10 points) Solve $3x^{\left(\frac{2}{3}\right)} = 24$. Simplify your answer(s) completely. Consider real solutions only.

34. (5 points) The expression $-3(x-2)^2-5$ represents a parabola. Does the parabola have a maximum or a minimum? Explain how you know. What are the coordinates of the maximum or minimum? What are the domain and range of the parabola?

maximum or minimum: _____

because:

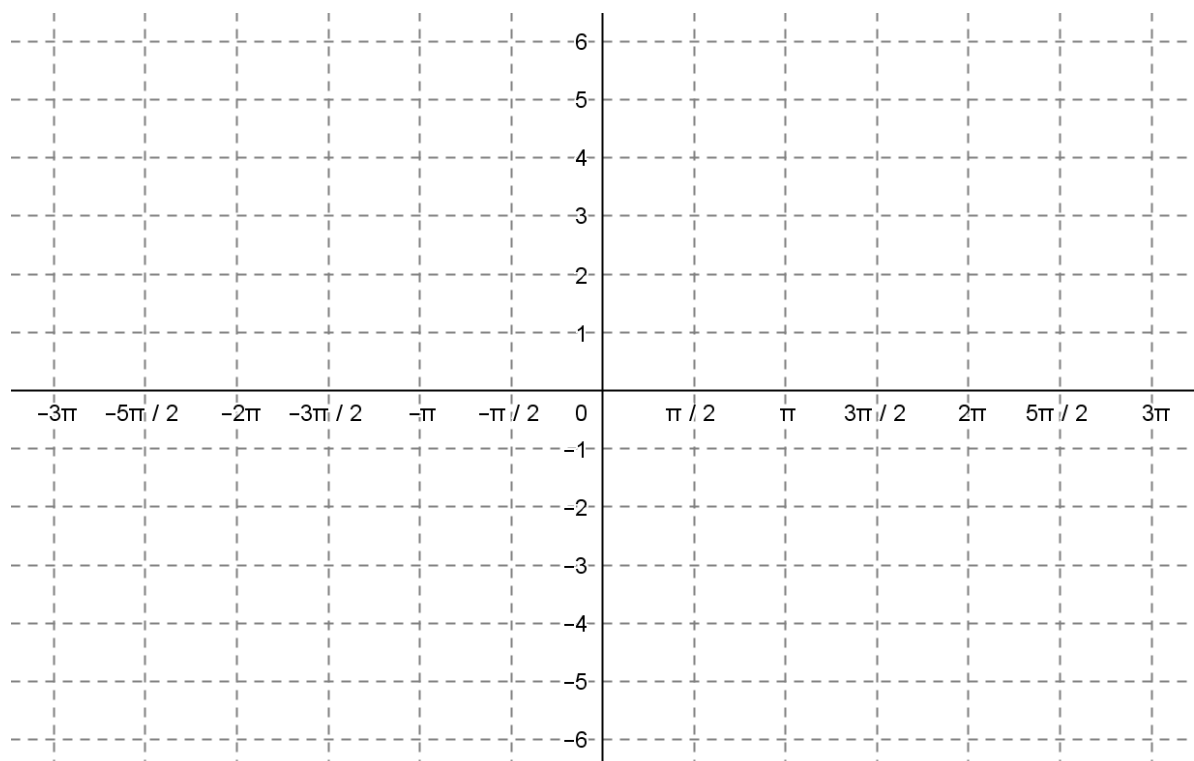
coordinates: _____

domain: _____

range: _____

35. (5 points) Solve $3e^{5x+1}=7$. Simplify your answer(s) completely. Consider real solutions only.

36. (5 points) Accurately sketch at least two periods (cycles) of the graph of $\cos\left(x - \frac{1}{2}\pi\right)$. First show the base function, then show the transformation. List the amplitude, period, and phase shift.



amplitude: _____

period: _____

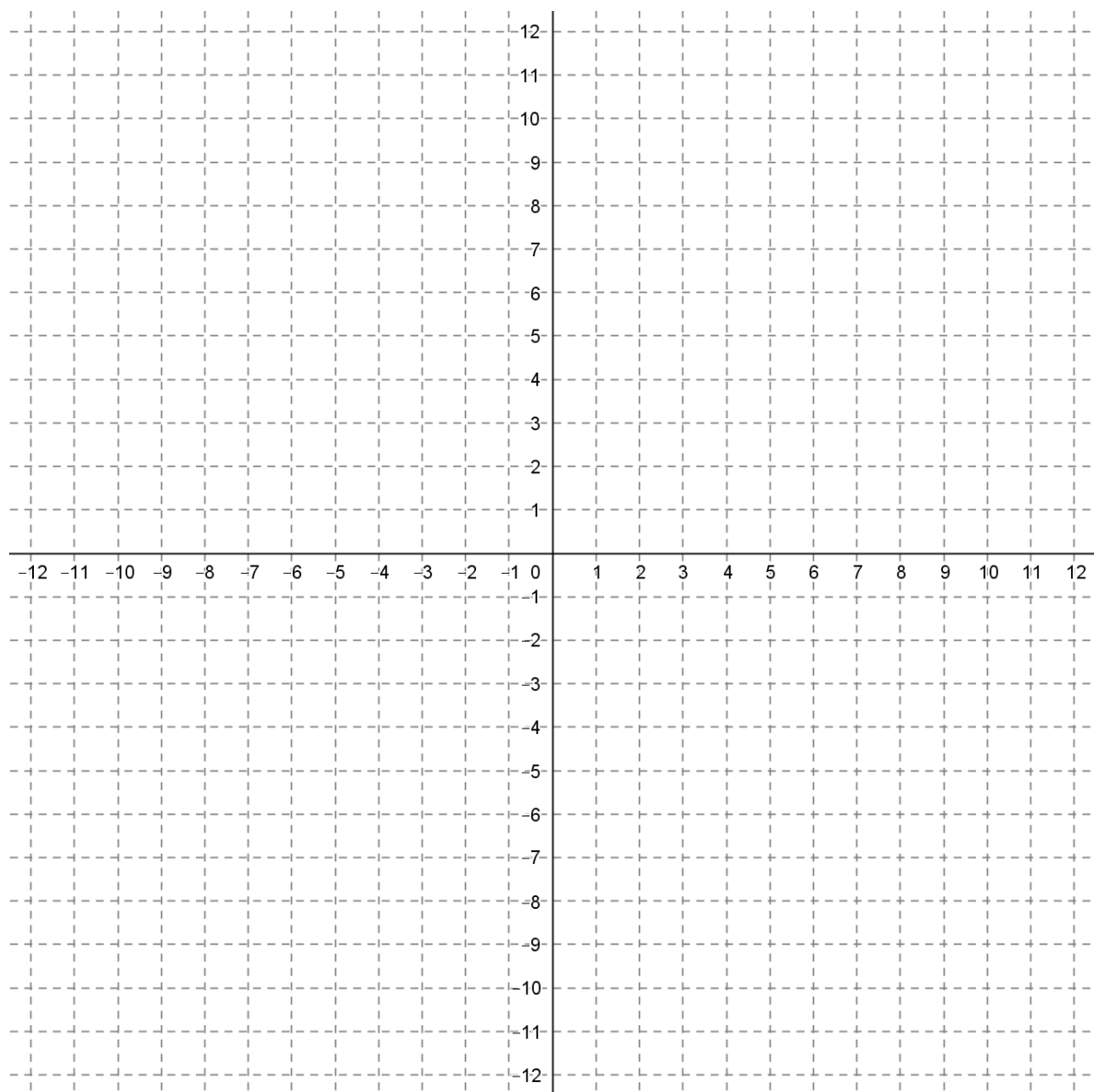
phase shift: _____

37. (5 points) Solve $2\log_3(x+1)=4$. Simplify your answer(s) completely. Consider real solutions only.

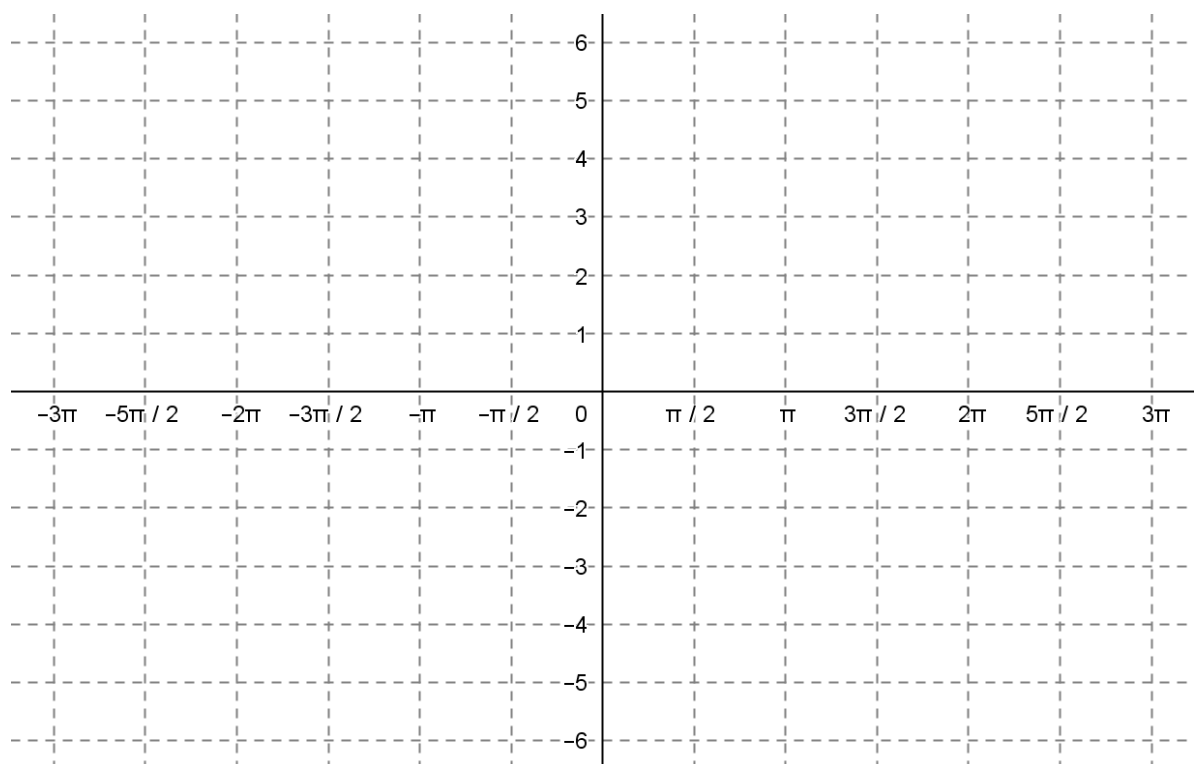
38. (5 points) Simplify the expression $\sqrt[3]{32x^3}\sqrt[3]{108x^5}$ completely. Give your answer with no rational (fractional) exponents.

39. (5 points) Simplify the expression $\frac{x+1}{2-\frac{1}{x+1}}$ completely.

40. (5 points) Accurately sketch the function $f(x) = -(x+4)^3 - 2$. First show the base function, then show the transformations. It may be helpful to create a table of x values and x^3 values.



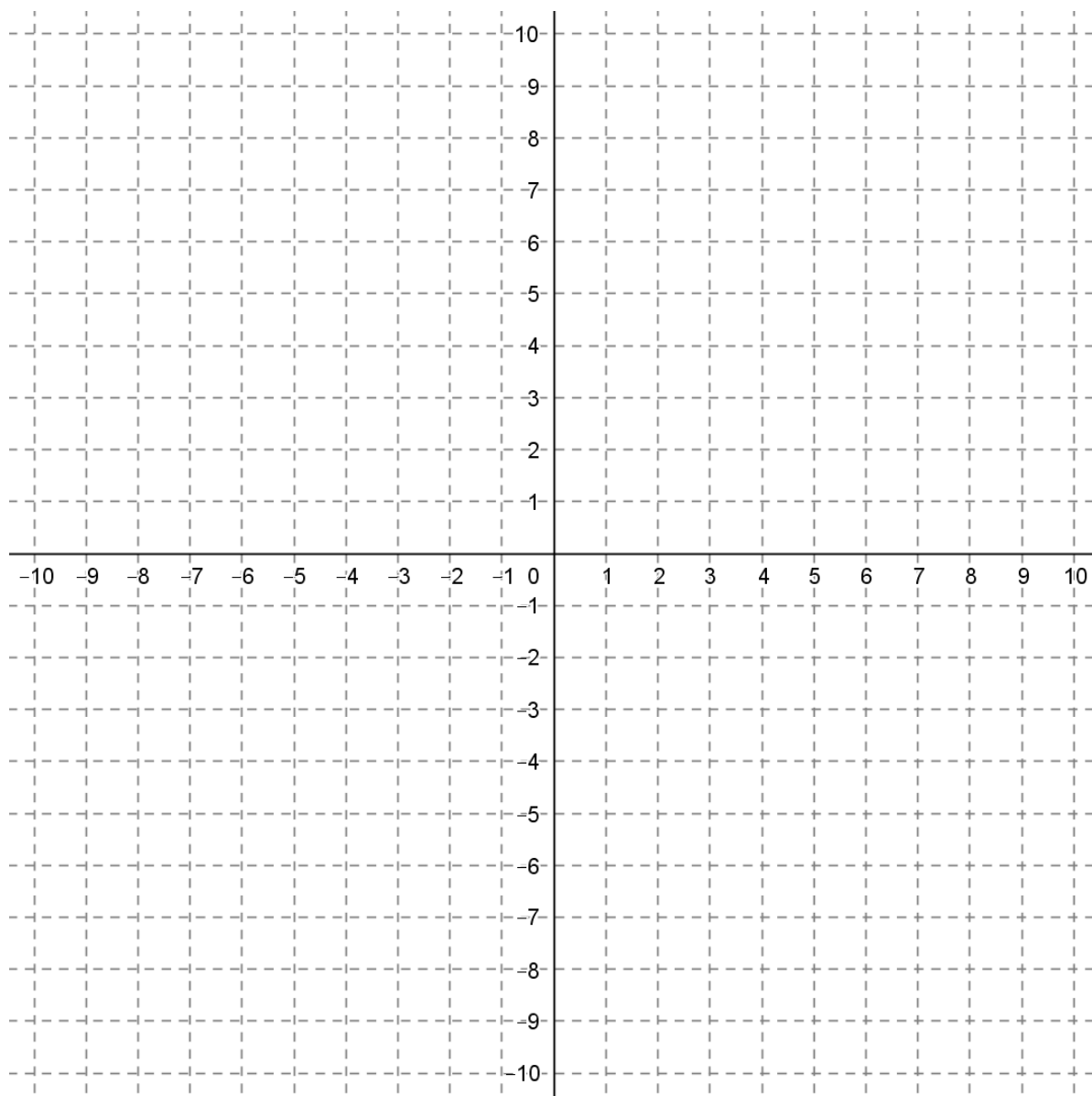
41. (5 points) Accurately sketch at least two periods (cycles) of the graph of $\tan(x)$. List the **equations** of at least two of the asymptotes.



equations of asymptotes: _____

42. (5 points) Solve $5x^5 + 15x^4 - 20x^3 = 0$ by factoring. Simplify your answer(s) completely.
Consider real solutions only.

43. (5 points) Accurately sketch the function $f(x) = \begin{cases} x^2 & x \leq 1 \\ 1 + 2x & x > 1 \end{cases}$. Determine $f(1)$.



$f(1) =$ 1

44. (5 points) Simplify the expression $\frac{-2x^2}{(3x)^{-3}}$ completely. Give your answer with no negative exponents and no rational (fractional) exponents.

45. (5 points) Solve $\frac{3}{4} - \frac{1}{6x} = \frac{1}{x}$. Simplify your answer(s) completely. Consider real solutions only.

46. (5 points) Solve the system of equations $\begin{cases} x+3y=5 \\ x-y=2 \end{cases}$. Simplify your answer(s) completely.
Consider real solutions only.

47. (5 points) Use long division to find $(2x^3 - 5x^2 + 5x - 6) \div (x - 2)$.

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