We discuss how to use spectral graph theory to count subgraphs of graphs where the subgraph counted is motivated by finite field versions of questions in geometric measure theory. One representative question is the following:

Let $E$ be a set in $F_q^d$ and $\alpha, \beta \in F_q^*$. How large does $E$ need to be to guarantee that there are four points $w, x, y, z \in E$ such that they form a rectangle of side lengths $\alpha$ and $\beta$, i.e.

(w-x)$\cdot(x-y) = 0,\ (x-y)\cdot(y-z) = 0,\ (y-z)\cdot(z-w) = 0,\ (z-w)(w-x) = 0,$ and

$$||w-x|| = ||y-z|| = \alpha,\ ||x-y|| = ||z-w|| = \beta. \tag{1}$$

We provide a general framework which answers this and other similar questions as a corollary. This is joint work with Thang Pham, Steven Senger, and Vu Thi Huong Thu.