Existence theory and propagation of oscillations for the system of viscoelasticity of strain-rate type

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I will review the existence and uniqueness theory for viscoelasticity of Kelvin-Voigt type with non-convex stored energies. The analysis is based on propagation of H^1-regularity for the deformation gradient of weak solutions in two and three dimensions assuming that the stored energy satisfies the Andrews-Ball condition, in particular allowing for non-monotone stresses. it turns out that weak solutions with deformation gradient in H^1 are in fact unique, providing a striking analogy to corresponding results in the theory of 2D Euler equations with bounded vorticity. On the opposite direction, while there is still existence of weak solution for initial data in L^2, there can be propagation of oscillations of the deformation gradient. A counterexample indicates that for non-monotone stress-strain relations in 1-d initial oscillations of the strain lead to solutions with sustained oscillations. Similar phenomena appear in several space dimensions associated with lack of rank-one convexity of the stored energy.
(joint work with K. Koumatos (U. of Sussex), C. Lattanzio and S. Spirito (U. of LAquila)).