Bimodules over Cartan MASAs, and Mercer's extension theorem

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Abstract: In a paper from 1991, Mercer asserts the following extension theorem:

> *For $i = 1, 2$, let $\mathcal{M}_i$ be a von Neumann algebra with separable predual, let $\mathcal{D}_i \subseteq \mathcal{M}_i$ be a Cartan subalgebra, and let $\mathcal{D}_i \subseteq \mathcal{A}_i \subseteq \mathcal{M}_i$ be a σ-weakly closed non-self-adjoint algebra which generates $\mathcal{M}_i$. If $\theta : \mathcal{A}_1 \to \mathcal{A}_2$ is an isometric algebra isomorphism such that $\theta(\mathcal{D}_1) = \mathcal{D}_2$, then there exists a unique $*$-isomorphism $\overline{\theta} : \mathcal{M}_1 \to \mathcal{M}_2$ such that $\overline{\theta}|_{\mathcal{A}_1} = \theta$.*

Mercer's proof relies on the *Spectral Theorem for Bimodules* of Muhly, Saito, and Solel (hereafter STB), which characterizes the σ-weakly closed $\mathcal{D}_i$-bimodules of $\mathcal{M}_i$ in terms of certain measure-theoretic data. Unfortunately, both proofs of STB in the literature contain gaps, and so the validity of STB, and therefore of Mercer's extension theorem, is unclear. In this talk, based on joint work with Jan Cameron (Vassar) and David Pitts (Nebraska), we prove Mercer's extension theorem under the additional hypothesis that θ is σ-weakly continuous. Our argument makes use of ideas from operator space theory, as well as a characterization of the Bures closed $\mathcal{D}_i$-bimodules of $\mathcal{M}_i$, and does not require that $\mathcal{M}_i$ have separable predual.