

TENSOR PRODUCTS OF $C(X)$ -ALGEBRAS OVER $C(X)$

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ABSTRACT: Let X be a compact Hausdorff space and $C(X)$ be the continuous complex-valued functions on X . A $C(X)$ -algebra is a unital C^* -algebra \mathcal{A} such that $C(X) \subseteq Z(\mathcal{A})$ (the center of \mathcal{A}). Given two $C(X)$ -algebras \mathcal{A} and \mathcal{B} , Blanchard shows that under certain conditions, the algebraic tensor product $\mathcal{A} \otimes_{C(X)} \mathcal{B}$ of \mathcal{A} and \mathcal{B} over $C(X)$ admits both a minimal C^* -norm and a maximal C^* -norm. In particular, this is true when X is hyperstonean, by a result of Giordano and Mingo.

In this talk, based on joint work with David Pitts, we survey the aforementioned theory and then use it to prove the following structural result for W^* -inclusions:

If \mathcal{M} is a W^ -algebra and $\mathcal{D} \subseteq \mathcal{M}$ is W^* -subalgebra, and if $(\mathcal{M}, \mathcal{D})$ admits a unique pseudo-expectation, then $\mathcal{D}^c = Z(\mathcal{D})$, where $\mathcal{D}^c = \mathcal{D}' \cap \mathcal{M}$ is the relative commutant of \mathcal{D} in \mathcal{M} .*

A key step in the proof is establishing that restriction is a bijection between the conditional expectations $C^*(\mathcal{D}, \mathcal{D}^c) \rightarrow \mathcal{D}$ and the conditional expectations $\mathcal{D}^c \rightarrow Z(\mathcal{D})$.