Multilevel Monte Carlo for inference

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For half a century computational scientists have been numerically simulating complex systems. Uncertainty is recently becoming a requisite consideration in complex applications which have been classically treated deterministically. This has led to an increasing interest in recent years in uncertainty quantification (UQ). Another recent trend is the explosion of available data. Bayesian inference provides a principled and well-defined approach to the integration of data into an a priori known distribution. The posterior distribution, however, is known only point-wise (possibly with an intractable likelihood) and up to a normalizing constant. Monte Carlo methods have been designed to sample such distributions, such as Markov chain Monte Carlo (MCMC) and sequential Monte Carlo (SMC) samplers. Recently, the multilevel Monte Carlo (MLMC) framework has been extended to some of these cases, so that approximation error can be optimally balanced with statistical sampling error, and ultimately the Bayesian inverse problem can be solved for the same asymptotic cost as solving the deterministic forward problem. This talk will concern the recent development of multilevel SMC (MLSMC) samplers [1] and the resulting estimators for standard quantities of interest as well as normalizing constants [2]. The methods have been applied successfully to nonlocal equations [5], which are used to model anomalous diffusion and fractures in materials. MLMC data assimilation methods have also been developed, which combine dynamical systems with data in an online fashion. Examples are ML particle filters [4] and ensemble Kalman filters [3].

References

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