Geometric classification of graph C*-algebras

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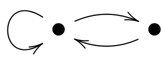
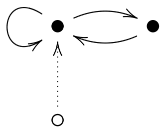
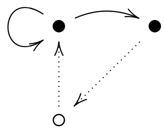
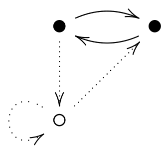
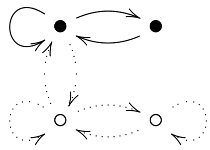
A construction originating with the work of Cuntz and Krieger allows the association of a C^*-algebra to any (countable) directed graph, in such a way that the properties of this functional analytical object reflects interesting aspects of the geometry and the combinatorics of the graph. One is naturally lead to the question of when two graphs will yield the same C^*-algebra, and when one chooses to work with the notion of stable isomorphism (or, which is the same, Morita equivalence) as the notion of equality of C^*-algebras, a very satisfactory answer is emerging in ongoing work inspired by the resolution of this question in the simple case fund by my former student Adam Sørensen.

Indeed, it is relatively easy to see that when, e.g., a graph



is given, then all the graphs

given by altering the original one by certain **moves** (source addition, delay, state splitting and Cuntz splice) as indicated will give the same C^*-algebra.

We have managed to prove that whenever two finite graphs yield the same stable C^*-algebra, one may transform one to the other by a finite number of such moves, not unlike the way knots are isotopic precisely when one may go from one to another by Reidemeister moves. The key to establishing this result is to recast it as a question of dynamics, in particular of flow equivalence of shifts of finite type, and appeal to a beautiful result by Boyle and Huang. One may even speculate that the matrix methods developed by Boyle and Huang can be amended to dealing with the full case of unital graph C^*-algebras.

This is joint work with Adam Sørensen (Copenhagen/Wollongong) and Efren Ruiz (Hawaii Hilo).