Abstract

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Topologically Mixing Tiling of $\mathbb{R}^2$ Generated by a Generalized Substitution

In my talk, I will present a large class of examples of topologically mixing self-similar tilings of the plane. Topologically mixing tiling dynamical systems were investigated by Kenyon, Sadun, and Solomyak in [2]. They studied one-dimensional tiling dynamical system generated by substitions on 2 letters. Given a substitution σ with transition matrix M_σ, they proved that if the lengths of the prototiles are irrationally related, and the eigenvalues λ_1, λ_2 have the property that $\lambda_1 > |\lambda_2| > 1$, then the one-dimensional tiling dynamical system is topologically mixing. They were, however, unable to extend there results beyond an alphabet with 2 letters or one-dimensional tilings.

The examples I have studied were first presented by Kenyon in [1]. He proved that any complex Perron number solving $\lambda^3 - p\lambda^2 + r\lambda + q = 0, p \geq 0, r, q \geq 1$ has a self-similar tiling. Solomyak in [3], studied general self-similar tilings. He was able to prove that any self-similar tiling dynamical system of $\mathbb{R}^d$, $d \in \mathbb{N}$ is never (measure theoretically) strong mixing. Solomyak was also able to prove that any self-similar tiling of $\mathbb{R}^2$ with a complex, non-Pisot similarity is weakly mixing. Soloymak used the construction of Kenyon to provide examples of weakly mixing tiling dynamical systems of the plane. However, the question as to whether any of these examples are topologically mixing tiling dynamical systems of $\mathbb{R}^2$ remained open. In my research, which was suggest by Solomyak, I have used techniques from [2], and [3] to prove that an infinite sub-collection of Kenyon's examples were topologically mixing. These are the first known examples of entropy-zero topologically mixing tiling dynamical systems of the plane.

References

[1] R. Kenyon. The construction of self-similar tilings. *Geom. Funct. Anal.*, 6(3):471–488, 1996.

[2] Richard Kenyon, Lorenzo Sadun, and Boris Solomyak. Topological mixing for substitutions on two letters. *Ergodic Theory Dynam. Systems*, 25(6):1919–1934, 2005.

[3] Boris Solomyak. Dynamics of self-similar tilings. *Ergodic Theory Dynam. Systems*, 17(3):695–738, 1997.