The pure extension property for discrete crossed products

Abstract: Let G be a discrete group acting on a unital C^*-algebra $\mathcal{A}$ by $*$-automorphisms. We show that the inclusion $\mathcal{A} \subseteq \mathcal{A} \rtimes_r G$ has the pure extension property (so that every pure state on $\mathcal{A}$ extends uniquely to a pure state on $\mathcal{A} \rtimes_r G$) if and only if G acts freely on $\widehat{\mathcal{A}}$, the spectrum of $\mathcal{A}$. The same characterization holds for $\mathcal{A} \subseteq \mathcal{A} \rtimes G$. This generalizes what was already known for $\mathcal{A}$ abelian. As an example, we show that the inclusion $\mathcal{O}_2 \subseteq \mathcal{O}_2 \rtimes \mathbb{Z}_2$ fails the pure extension property.