Faithfulness vs. Uniqueness, for Pseudo-Expectations

Abstract: A **pseudo-expectation** for a C^*-inclusion $\mathcal{A} \subseteq \mathcal{B}$ is a unital completely positive (UCP) map $\theta : \mathcal{B} \to I(\mathcal{A})$ such that $\theta|_{\mathcal{A}} = \mathrm{id}$, where $I(\mathcal{A})$ is Hamana's injective envelope of $\mathcal{A}$. Pseudo-expectations generalize conditional expectations, with the advantage that they always exist for any C^*-inclusion. An interesting open question about the pseudo-expectation space is the following:

> *If every pseudo-expectation for a C^*-inclusion is faithful, must there be a unique pseudo-expectation?*

After providing background and motivation for this question, we will show that it has an affirmative answer for C^*-inclusions $\mathcal{A} \subseteq \mathcal{A} \rtimes_r G$ arising from the action of a discrete group G on a unital C^*-algebra $\mathcal{A}$ by $*$-automorphisms.