CARTAN ENVELOPES

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Abstract. A regular inclusion is a pair $(\mathcal{C}, \mathcal{D})$ of C^*-algebras where $\mathcal{D} \subseteq \mathcal{C}$ is abelian and the set $\{v \in \mathcal{C} : v\mathcal{D}v^* \cup v^*\mathcal{D}v \subseteq \mathcal{D}\}$ has dense span in $\mathcal{C}$.

An important and well-studied class of regular inclusions are *Cartan inclusions*, which were introduced by Renault (building upon work of Kumjian). Renault makes a strong case that Cartan inclusions are the appropriate C^*-algebraic variant of a Cartan MASA in a von Neumann algebra. In addition, Cartan inclusions have numerous useful structural properties.

I will describe the notion of a *Cartan envelope* for a regular inclusion $(\mathcal{C}, \mathcal{D})$; the idea is that the Cartan envelope should be the smallest Cartan pair generated by $(\mathcal{C}, \mathcal{D})$. In Part 1, I characterize which *unital* regular inclusions have Cartan envelopes, discuss the construction, uniqueness and minimality properties for the Cartan envelope, and describe a groupoid model for the Cartan envelope.

In the non-unital setting, the definition of the Cartan envelope is more subtle; nevertheless, it is still possible to define the Cartan envelope. It turns out that for inclusions having the approximate unit property, Cartan envelopes behave well with respect to the unitization process, however establishing this fact is not simply a matter of adjoining a unit and using the unital case. In Part 2, I discuss the non-unital case, give some examples illustrating some of the difficulties, and also show that in the presence of the approximate unit property, one can obtain existence, uniqueness and minimality of Cartan envelopes in the non-unital case.