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A simple measurement of the relative efficiency of human locomotion

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Abstract

Elementary formulas for gravitational potential energy, heat capacity, and Newton's law of cooling are used to measure and compare the efficiencies of a person walking uphill and downhill. Good agreement is found for their ratio with more sophisticated experiments in the literature made by measuring respiration to determine metabolic power. A simple argument explains why human muscles are less than 50% efficient at converting chemical into mechanical energy.

Human beings can be modelled as engines insofar as they use their muscles to accomplish physical tasks. The net energy input from eating, drinking, and breathing is taken to be chemical energy E_{chem}. The net energy output from muscular activity will be written as work W which is equal to a person's change in gravitational potential energy ΔU when walking up or down a hill at a constant speed. Assuming the person is in an environment where the ambient temperature is lower than body temperature, there must be a net heat output Q to the surroundings. Further if the body maintains equilibrium on average, then energy conservation requires $E_{\text{chem}} = W + Q$. When resting (i.e. no muscular work is done other than motion of the diaphragm for breathing so that $W = 0$) then $E_{\text{chem rest}} = Q_{\text{rest}}$. For the purposes of analysing human locomotion, redefine E_{chem} and Q to be the excesses beyond these resting values. Ratios of the energy excesses can then be used to define an efficiency ε or coefficient of performance κ. Keeping in mind that W is negative if a person is traveling downhill [1], one choice is $\kappa \equiv Q/W$ which can be recast into other ratios of interest such as $\varepsilon \equiv W/E_{\text{chem}} = W/(W+Q) = 1/(1+\kappa)$, analogous to the relation between the coefficient of performance κ of a Carnot heat pump and the efficiency ε of a Carnot engine operating between the same two temperature reservoirs [2].

Carefully controlled measurements of human volunteers by exercise physiologists have resulted in values of $\varepsilon = 0.25 \Rightarrow \kappa = 3.0$ for walking uphill and $\varepsilon = -1.2 \Rightarrow \kappa = -1.8$ for walking downhill [3]. These values are used to sketch the energy flow diagrams in figure 1. In these experiments by Margaria, the subjects wore masks so that the rates of oxygen uptake and carbon dioxide exhalation could be measured and converted to metabolic power [4]. We sought a simpler experimental technique that uses only principles from elementary physics to motivate interest by life science students taking an introductory course [5]. Our idea was to use a Vernier stainless steel temperature probe consisting of a 10 cm-long smooth metal rod connected to a thermistor with a resolution of 0.03 °C near room temperature [6]

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where data is logged twice a second by a battery-powered LabQuest 2. We initially tried to make measurements by running up and down either outside on a hill or inside on a five-story spiral staircase, but variations in the ambient conditions (especially the air temperature) led to noisy data. Therefore, we instead employed a treadmill in a fixed location. (In the reference frame of the walking person, an inclined treadmill is equivalent to an actual hill.) Ordinary treadmills do not have downhill settings, and so we used a horizontal machine (having a length of 36 inches between its front and back legs) and blocked up either the front or back end by 3 inches, corresponding to an angular tilt of $\theta = \sin^{-1}(3/36) = 4.8°$, positive for the uphill case and negative for downhill. The treadmill was run at a comfortable speed of $\upsilon = 2.4$ mph $= 1.1$ m s^{-1} (corresponding to a brisk walking pace) for safety reasons, particularly when used in the unfamiliar downhill orientation. The rate of work done $\dot{W} = \Delta U/\Delta t$ is $mg\upsilon\sin\theta$ for a person of mass m where Earth's surface gravitational field strength is $g = 9.8\,\mathrm{m\,s^{-2}}$. The initial rate of heating of a person's body at temperature T is $\dot{Q} = mc\Delta T/\Delta t$. Here c is the specific heat which should be comparable to that of water, $4180\,\mathrm{J \cdot kg^{-1} \cdot °C^{-1}}$. Student author Comeford was the treadmill subject and as a reasonably fit military officer, his actual specific heat [7] is $c \approx 3570\,\mathrm{J \cdot kg^{-1} \cdot °C^{-1}}$. Consequently, we predict

$$\kappa = \frac{\dot{Q}}{\dot{W}} = \frac{c\Delta T/\Delta t}{g\upsilon\sin\theta}, \tag{1}$$

which using the numbers given above[4] for uphill walking implies an average body temperature increase of only $\Delta T = 0.14\,°\mathrm{C}$ in 3 min. That is the approximate maximum amount of time the student could perform the exercise in athletic gear before the rise in his body temperature began to level off. Unfortunately, this value of ΔT is too close to the resolution limit of our thermistor to measure accurately. However, by placing the temperature probe inside his compression shorts (partly insulated from the cool ambient air) in direct contact with the thigh (nearest the leg muscles doing most of the mechanical work), a readily

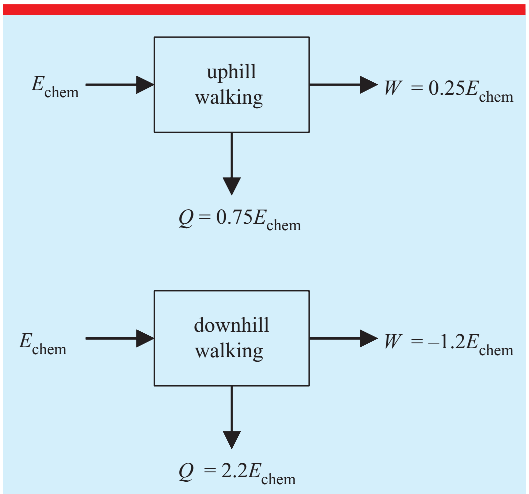

Figure 1. Energy inputs and outputs when modelling a person walking up or down a hill as a thermodynamic engine cycle, using the efficiency parameters of Margaria [3].

measurable temperature rise exceeding 1.5 °C was obtained, as plotted in figure 2. The right-hand side of equation (1) then needs to be multiplied by a factor f that represents the fraction of the mass of the student's (partly insulated) thigh that is (initially) heating up relative to the student's overall body mass. To eliminate this factor, take the ratio of κ for uphill and downhill motion to get the *relative* coefficient of performance

$$\left|\frac{\kappa_{\text{uphill}}}{\kappa_{\text{downhill}}}\right| = \frac{\dot{T}_{\text{initial uphill}}}{\dot{T}_{\text{initial downhill}}}, \tag{2}$$

where $\dot{T}_{\text{initial}}$ is the rate of increase in temperature (in °C/s) of the thigh just after motion commences (before any cooling to the surroundings occurs).

To find the initial rate of temperature increase, use Newton's law of cooling to model the heat loss between the thigh and the surroundings. That law implies an exponentially saturating temperature change [8] given by

$$T = T_{\mathrm{f}} - (T_{\mathrm{f}} - T_{\mathrm{i}})\mathrm{e}^{-t/\tau}, \tag{3}$$

where the temperature of the thigh is initially T_{i} but levels off finally to T_{f} and where τ is a thermal transfer time constant that depends on the total rate of cooling (by convective and radiative cooling of the thigh to the surroundings, and by thermal conduction via the flesh and blood from the thigh to other parts of the body). Extrapolated to zero time, the slope of equation (3) is

$$\dot{T}_{\text{initial}} = (T_{\mathrm{f}} - T_{\mathrm{i}})/\tau. \tag{4}$$

[4] Margaria's values of ε are for steeper inclines than the 1/12 slope we are using, but our model is only approximate in any case.

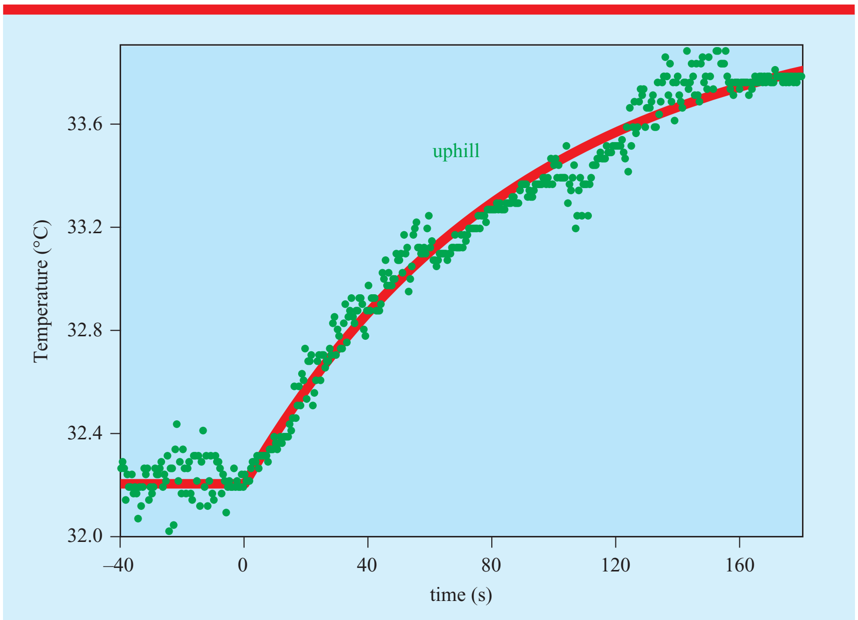

Figure 2. Temperature versus time data for walking uphill on a treadmill. The zero of time was adjusted horizontally to coincide with the start of the exercise. Time was allowed for the temperature probe to equilibrate with the skin temperature under the subject's shorts to an average reading of 32.20 °C (indicated by the red horizontal line at negative times, corresponding to a temperature intermediate between ordinary body temperature of 37 °C and room temperature of 22 °C). The red curve for positive times is a fit to equation (3).

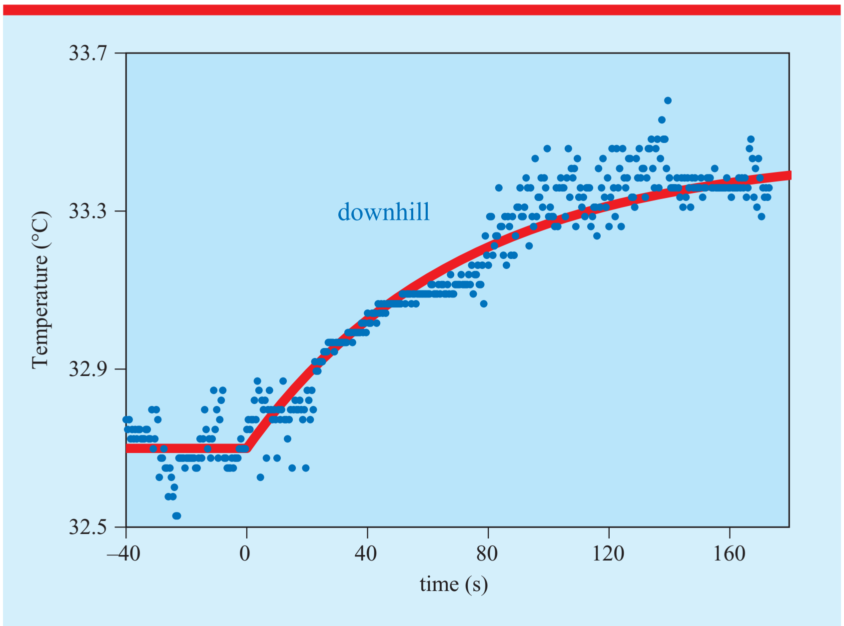

Figure 3. Temperature versus time data for walking downhill on a treadmill. The zero of time was adjusted horizontally to coincide with the start of the exercise. Time was allowed for the temperature probe to equilibrate with the skin temperature under the subject's shorts, as indicated by the red horizontal line at negative times. The red curve for positive times is a fit to equation (3). The horizontal range of times is the same as in figure 2, but the vertical range of temperatures is smaller here than in figure 2, because one's body warms up less when traveling downhill than uphill over a given time interval (for the same treadmill slope and speed).

The average initial temperatures are indicated by the horizontal red lines in figures 2 and 3 at negative times. Fitting equation (3) to the measured data at positive times gives the red curves in those figures. For the uphill case, we find $T_i = 32.20\,°C$, $T_f = 34.05\,°C$, and $\tau = 90\,s$. Equations (1) and (4) then predict $\kappa_{uphill} = 81$. To match Margaria's value of 3.0, this result implies $f \approx 4\%$. For the downhill case, we get $T_i = 32.70\,°C$, $T_f = 33.45\,°C$, and $\tau = 70\,s$, which implies $\kappa_{downhill} = -42$. (This coefficient of performance is negative because the angle θ is negative when going downhill.) That matches Margaria's value of -1.8 for approximately the same value of f. These results show that simple physics ideas can indeed be used to model the energetics of human locomotion. The key difference between figures 2 and 3 is that $\Delta T \equiv T_f - T_i$ equals 1.85 °C when traveling uphill but only 0.75 °C when going downhill. By repeating the data collection on four different days (when ambient conditions were changed, and the student was in a different state of readiness), the standard deviation in the raw κ values reported here is found to be ± 26. We encourage teachers to further explore these ideas experimentally and theoretically with their students to see if they can reduce this error bar. Analysis of muscle work is a topic of active interest in sports science [9] and biophysics [10].

In closing, when walking downhill, the best one could hope to achieve is to convert all the lost gravitational potential energy directly into heat without any bodily expenditure of chemical energy, which would imply $\kappa_{downhill} = -1$. Margaria's value of -1.8 thus seems reasonable, because it takes some muscular effort to keep from tumbling downhill. Furthermore, we know from everyday experience that we get hotter when walking up a hill than down the same hill at the same speed. Consequently $\kappa_{uphill} > |\kappa_{downhill}|$ and so even in the ideal case one must have $\kappa_{uphill} > 1$ and thus $\varepsilon_{uphill} < 1/2$ which implies that the chemical-to-mechanical conversion efficiency of our body for climbing a hill must be less than 50%, thereby putting Margaria's value of 25% into perspective.

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Professor Carl E Mungan is the research advisor for Midshipman Austin R Comeford. This project was a detour from our work on laser spectroscopy of dye-doped polymers, motivated by e-mail discussions initiated by Professor Nathaniel R Greene.

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